

SUBCOURSE
EN0591

EDITION
C

US ARMY ENGINEER CENTER AND SCHOOL

SURVEYING I



"LET US TRY"

THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT
ARMY CORRESPONDENCE COURSE PROGRAM

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SURVEYING I

Subcourse EN0591

EDITION C

United States Engineer Center and School
Fort Leonard Wood, Missouri 65473

7 Credit Hours

Edition Date: November 1997

SUBCOURSE OVERVIEW

This subcourse is designed to give soldiers a practical knowledge of surveying fundamentals and equipment, as well as a review of some of the mathematics needed in surveying operations.

The prerequisite for this course is a basic knowledge of mathematic principles, to include multiplication, division, fractions, and decimals or the completion of the Army Correspondence Course Program (ACCP) Missile & Munitions Subcourse (MM) 0099.

This subcourse reflects the doctrine that was current when it was prepared. In your own work situation, always refer to the latest publication.

At Appendix D, you will find an English/metric conversion chart.

Unless otherwise stated, the masculine gender of singular pronouns is used to refer to both men and women.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use math, geometry, and trigonometry in surveying calculations and be able to identify and understand the uses of surveying equipment.

CONDITION: You will be given the material in this subcourse and an ACCP examination response sheet.

STANDARD: To demonstrate competency of this task, you must achieve a minimum of 70 percent on the subcourse examination.

TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
Subcourse Overview	i
Administrative Instructions	iv
Grading and Certification Instructions.....	iv
Lesson 1: Introduction to Surveying.....	1-1
Part A: Surveying Career Field.....	1-2
Part B: General.....	1-2
Part C: Terminology.....	1-5
Part D: Fieldwork.....	1-8
Part E: Surveying Parties.....	1-12
Part F: Surveys.....	1-12
Part G: Control Surveys	1-15
Part H: Professional Societies and Manufacturers.....	1-16
Practice Exercise.....	1-19
Answer Key and Feedback.....	1-20
Lesson 2: Ratios, Proportions, Roots, and Powers.....	2-1
Part A: Ratios and Proportions	2-2
Part B: Roots and Powers.....	2-6
Practice Exercise.....	2-9
Answer Key and Feedback.....	2-10
Lesson 3: Geometry	3-1
Part A: Lines	3-1
Part B: Angles.....	3-2
Part C: Triangles	3-5

	<u>Page</u>
Part D: Postulates	3-7
Part E: Theorems for Lines, Angles, and Triangles.....	3-8
Part F: Review of Geometry.....	3-21
Practice Exercise.....	3-23
Answer Key and Feedback.....	3-24
Lesson 4: Trigonometry	4-1
Part A: Angles.....	4-1
Part B: Trigonometric Fundamentals.....	4-7
Practice Exercise.....	4-33
Answer Key and Feedback.....	4-34
Lesson 5: Surveying Equipment.....	5-1
Part A: Universal Surveying Instruments.....	5-1
Part B: Field Equipment.....	5-6
Part C: Associated Surveying Equipment	5-8
Practice Exercise.....	5-25
Answer Key and Feedback.....	5-26
Examination.....	E-1
Appendix A: List of Common Acronyms	A-1
Appendix B: Recommended Reading List.....	B-1
Appendix C: Natural Trigonometric-Functions Tables	C-1
Appendix D: English/Metric Conversion Chart	D-1

GRADING AND CERTIFICATION INSTRUCTIONS

Examination: This subcourse contains a multiple-choice examination covering the material in the five lessons. After studying the lessons and working through the practice exercises, complete the examination. Mark your answers in the subcourse booklet, then transfer them to the ACCP examination response sheet. Completely black out the lettered oval that corresponds to your selection (A, B, C, or D). Use a number 2 pencil to mark your responses. When you complete the ACCP examination response sheet, mail it in the preaddressed envelope provided with the subcourse. You will receive an examination score in the mail. You will receive seven credit hours for successful completion of this examination.

LESSON 1

INTRODUCTION TO SURVEYING

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn the fundamentals of surveying.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn the fundamentals of surveying.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Surveying is a science that deals with determining the relative position of points on or near the earth's surface. These points may be needed for construction to locate or lay out roads, airfields, and structures of all kinds or for cultural, hydrographic, or terrain features for mapping. In the military, these points may be used as target reference points for artillery. The horizontal position of these points is determined from the distances and directions measured in the field. The vertical position is computed from the differences in elevations, which are measured directly or indirectly from an established point of reference or datum.

Surveying is a basic subject in the training of any civil engineer regardless of any ultimate specialization. Its application requires skill as well as knowledge of mathematics (which will be covered later in this correspondence course), physics, drafting and, to some extent, astronomy. This chapter will give you an overview of surveying in general, with emphasis on those areas affecting the duties of a construction surveyor.

Surveying involves fieldwork and office work. Fieldwork consists of taking measurements, collecting engineering data, and testing material. Office work consists of analyzing and computing field data and drawing the necessary sketches.

As your experience increases, you will realize that accuracy in surveying is very important because the results of the surveys are the basis of other factors affecting sound decisions in engineering practice.

PART A - SURVEYING CAREER FIELD

1-1. Career Progression. The normal line of progression for a technical engineering specialist (military occupational specialty (MOS) 51T) and a topographic surveyor (MOS 82D) are discussed in Army Regulation (AR) 611-201.

a. The following listed subcourses for surveyor training are available from the United States (US) Army Engineer School, Fort Leonard Wood, Missouri:

- EN 0591 Surveying I (Mathematics and Surveying Principles).
- EN 0592 Surveying II (Plane-Surveying Operations).
- EN 0593 Surveying III (Topographic and Geodetic Surveying).
- EN 0594 Surveying IV (Construction Surveying).

b. Subcourses EN 0591, EN 0592, and EN 0594 are intended to provide the theoretic training required for a construction surveyor (MOS 51T20). Subcourses EN 0591, EN 0592, and EN 0593 are intended to provide the theoretic training required for a topographic surveyor (MOS 82D20/30).

c. Surveyor training is not complete without some practical work in the field. This should occur after the student has had experience handling and using at least one of each of the more common types of instruments and equipment described in this lesson. Students enrolled in these subcourses should be assigned to an organization or unit that does construction or topographic surveying. If they are not, they must acquire on-the-job training while studying these subcourses.

PART B - GENERAL

1-2. Purpose of a Survey. The earliest application of surveying was for establishing land boundaries. Surveying has also branched out to many fields that parallel the advancement of civil engineering and civilization. Surveyors may be called upon to appear in court to substantiate definite locations of various objects, such as those involved in major traffic accidents, maritime disasters, or even murder cases. Surveying methods may be the same, but their purposes are varied. Generally, surveys are conducted-

- For subdividing or establishing boundaries of land properties. (NOTE: As the value of real estate increases, the demand for good land surveyors also increases.)

- For studying the actual construction of public or private works. (NOTE: As a construction surveyor in the Army, this is the type of surveying you will conduct.)
- On major operations with a higher order of accuracy that only government agencies are equipped to handle; for example, hydrography and the proposed project to tie the world into one triangulation network using earth-orbiting satellites.

Again, although these surveys are for various purposes, the basic operations are the same—they involve measurements and computations or, basically, fieldwork and office work.

1-3. Duties of the Construction Surveyor. In support of construction activities, the surveyor obtains reconnaissance and preliminary data that are necessary at the planning stage. During the construction phase, the surveyor supports the effort as needed. Typical duties of the construction surveyor include—

- Determining distances, areas, and angles.
- Establishing reference points for both horizontal and vertical control.
- Setting stakes or marking lines, grades, and principal points.
- Determining profiles of the ground along given lines (centerlines and/or cross-section lines) to provide data for cuts, fills, and earthwork volumes.
- Laying out structures, culverts, and bridge lines.
- Determining the vertical and horizontal placement of utilities.

1-4. Relative Location and Position. The location of a point on the earth can only be described in terms of the relative location or position of the point with reference to another point. This relative location or position of a point on the earth's surface and the corresponding point on a map may be described in terms of a system of coordinates. Coordinates are quantities that designate the position of a point in relation to a given reference frame. Telling someone that the Main post exchange (PX) is two blocks north of Main Street and three blocks east of Broadway is using coordinates. A point may also be identified by its latitude and longitude and its distance and direction from another point.

a. Coordinates. Coordinates are often used with a grid, which is a network of uniformly spaced straight lines intersecting at right angles. Figure 1-1, page 1-4, shows a grid. The reference frame consists of horizontal and vertical baselines. Each baseline in this example is divided into units of measurement, and each unit is further divided into tenths. The direction of the vertical baseline is called north, and the direction of the horizontal baseline is called east. The intersection of the baselines is called the origin and has a coordinate value of zero-zero. The dot is located 2 units plus three-tenths of a unit more, or 2.3 units, above the horizontal baseline. The dot is also 1.1 units east of the vertical baseline. The coordinates of this grid is 2.3 north and 1.1 east.

b. Latitude and Longitude. Latitude and longitude is the position of any point in relation to where the north-south (NS) line (the zero or Greenwich meridian) intersects the east-west (EW) line (the zero parallel or the earth's equator). This location method is known as the geodetic-coordinates method.

c. Distance and Direction. Another location method is distance and direction. In the following examples, the location is given in terms of the point's direction and the distance from the reference point:

- Example 1: A certain point is located 15 miles southwest of the center of Minneapolis.
- Example 2: In Figure 1-2, standing at the corner of the garage, the tree is 45° clockwise from the edge of the driveway and 50 steps away from the garage.

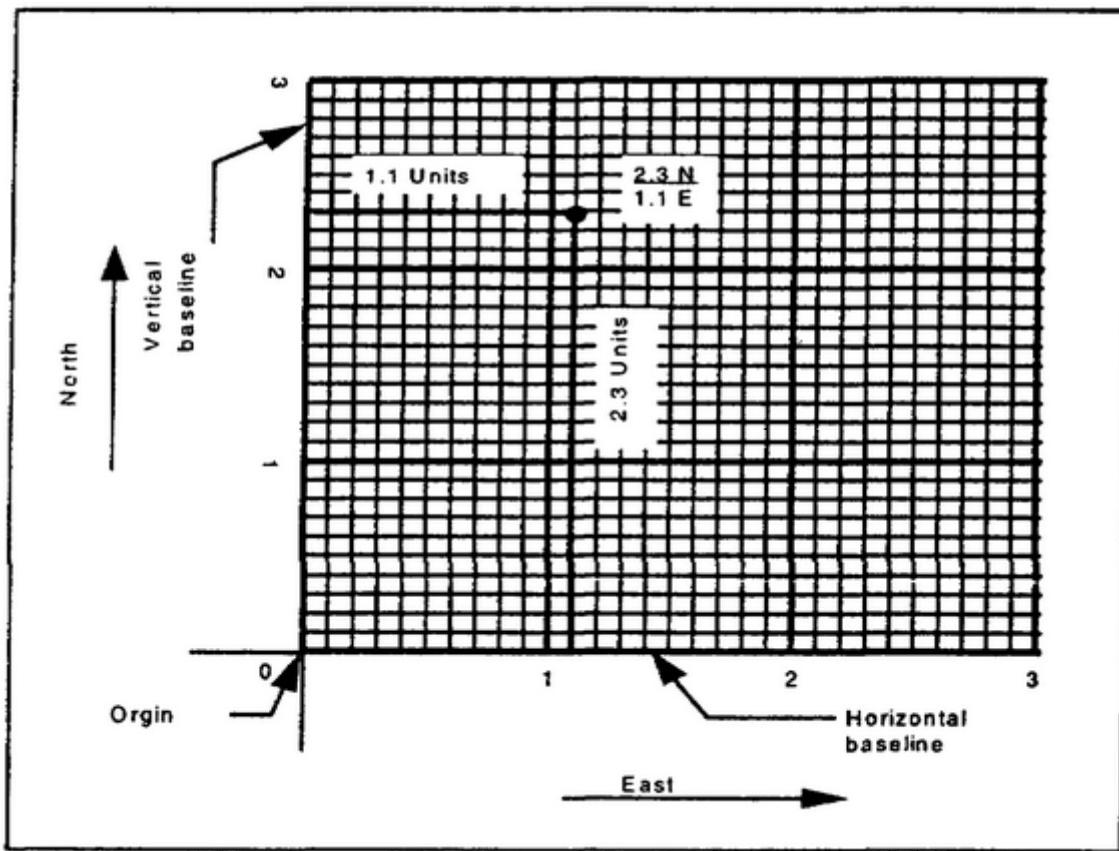


Figure 1-1. Coordinates of a point on a grid

1-5. Plane Surveying. The branch of surveying in which the mean surface of the earth is considered a plane surface is generally referred to as plane surveying. In plane surveying, the earth's curvature is neglected, and computations are made using the formulas of plane geometry and trigonometry. In general, plane surveying is applied to surveys of land areas and boundaries (land surveying) where the areas are of limited extent. Plane surveying is also used when the required accuracy is so low that corrections for the effect of curvature would be negligible as compared to the errors of observations. For small areas, precise results may be obtained with plane-surveying methods, but the accuracy and precision of such results will decrease as the area surveyed increases in size. Generally, plane

surveying is done for the location and construction of highways, railroads, canals, and landing strips.

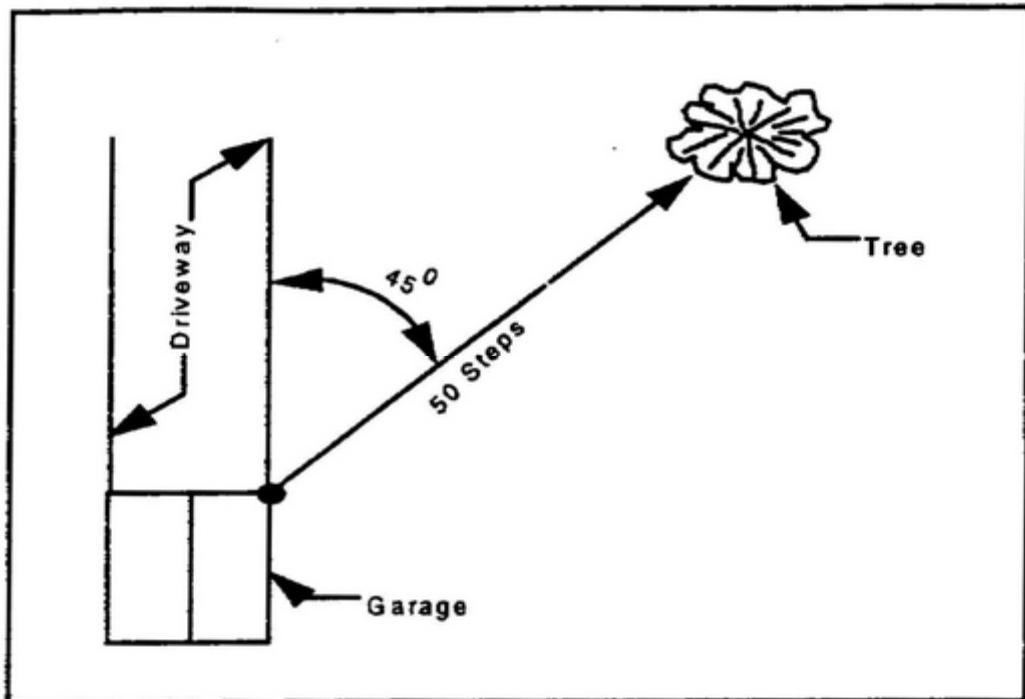


Figure 1-2. Distance and direction

Leveling operations are usually considered a part of plane surveying. The effect of the earth's curvature is automatically taken into account when using standard procedures to determine elevations. Elevations are referred to a certain datum, which is a plane tangent at any point of the earth's surface and normal to the plumb line at that point. This datum is normally taken at an imaginary plane tangent to the surface of mean sea level. Figure 1-3, page 1-6, illustrates the fact that when this datum plane is extended 10 miles out from the point of tangency, the vertical distance (elevation) of the plane above the surface represented by mean sea level is 67 feet, and at a distance of 100 miles, this elevation becomes 6,670 feet. For this reason, the earth's curvature cannot be neglected as a factor in taking even rough elevations. Later, you will learn the importance of maintaining, as much as possible, a balanced distance between the foresight and backsight in leveling operations.

PART C - TERMINOLOGY

1-6. Surveying Terms. The following surveying terms are just a few of the technical terms that you will be using. You will learn most of them in actual practice and through usage.

- *Agonic line*-The line along which the magnetic declination is zero.
- *Angle of inclination*-A vertical angle of elevation or depression.
- *Azimuth*-The angle to a line of sight, measured clockwise from any meridian and range from 0° to 360° .

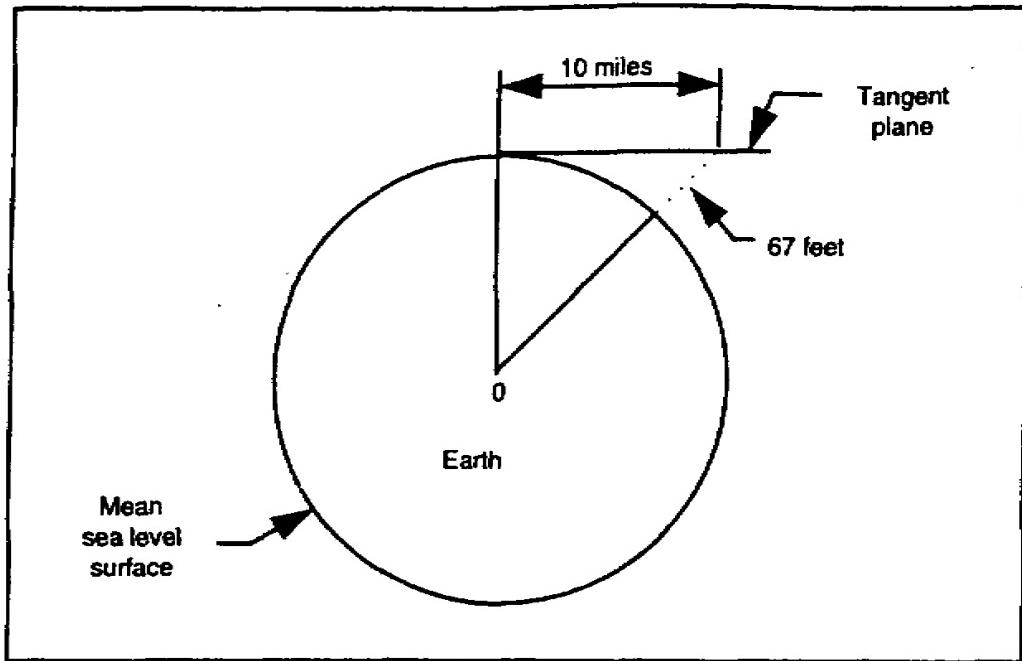


Figure 1-3. The effect of the earth's curvature

- *Blaze*-A mark made on the trunk of a standing tree by chipping off a spot of bark with an ax. It is used to indicate a trail, a boundary, a location for a road, a tree to be cut, and so on.
- *Bubble axis (level vial)*-The horizontal line tangent to the upper surface of the centered bubble, which lies in the vertical plane through the longitudinal axis of the bubble tube.
- *Calibration*-The process of standardizing a measuring instrument by determining the deviation from a standard so as to ascertain the proper correction factors.
- *Collimation*-The act of adjusting the line of sight of a telescopic surveying instrument to its proper position relative to the other parts of the instrument.
- *Collimation line*-The line through the second nodal point of the objective (object glass) of a telescope and the center of the reticle. It is also referred to as the line of sight, sight line, pointing line, and the aiming line of the instrument. The center of the telescope reticle can be defined by the intersection of crosshairs or by the middle point of a fixed vertical wire or a micrometer wire in its mean position. In a leveling instrument, the center of the reticle may be the middle point of a fixed horizontal wire.
- *Contour*-An imaginary level line (constant elevation) on the ground surface; it is called a contour line on a corresponding map.
- *Datum*-Any numeric or geometric quantity that serves as a reference or base for other quantities. It is described by such names as geodetic, leveling, North American, or tidal datum, depending on its purpose when established.

- *Elevation*-The vertical distance of a point above or below an established or assumed point (datum) on a level surface.
- *Geodetic datum*-Datum that forms the basis for the computation of horizontal-control surveys in geodetic surveying. It consists of five quantities: the latitude and the longitude of an initial point, the azimuth of a line from this point, and two constants necessary to define the terrestrial spheroid.
- *Grade (gradient)* -The rate of rise and fall or slope of a line; generally expressed in percent or as a ratio.
- *Horizontal angle*-The angle formed by two intersecting lines on a horizontal plane.
- *Horizontal distance*-A distance measured along a level line. It is commonly thought of as the distance between two points. The distance may be measured by holding a tape horizontally or by measuring the inclined distance between the points. However, the inclined distance is always reduced to its horizontal length.
- *Horizontal line*-A straight line perpendicular to a vertical line at a given point.
- *Horizontal plane*-A plane tangent to a level surface (also called plane of the horizon).
- *Instrument adjustment*-Adjusting the parts of an instrument to obtain the highest practical precision. For example, field adjustments of theodolites include adjusting the bubble tube, circular bubble, line of sight, horizontal axis, telescope bubble tube, vertical circle, and optical plummet.
- *Isogonic line*-An imaginary line or a line on a map joining points on the earth's surface at which the magnetic declination is the same.
- *Legend*-A description, explanation, table of symbols, and so on printed on a map or chart for a better understanding and interpretation of it.
- *Level surface*-A surface that is parallel with the spherical surface of the earth, such as a body of still water.
- *Leveling datum*-A level surface to which elevations are referred. Generally, the adopted datum for leveling in the US is the mean sea level.
- *Measured angles*-Angles that are either vertical or horizontal.
- *Parallax*-An error in sighting that occurs when the objective and/or the crosshairs of a telescope are improperly focused. In testing the focusing of a telescope, the head of the observer must move from side to side or up and down while sighting through the eyepiece. Any apparent movement of the crosshairs in relation to the object image means that parallax is present.

- *Plumb line*-A line (as a cord) that has at one end a weight (as a plumb bob) and is used to determine verticality.
- *Reticle*-A scale on transparent material (as in a telescope) used especially for measuring or aiming.
- *Station*-The location of a definite point on the earth's surface that has been determined by surveying methods. It may be a point on a traverse over which an instrument is set up or a length of 100 feet measured on a given line-broken, straight, or curved.
- *Traverse*-A sequence of lengths and directions of lines between points on the earth, obtained by or from field measurements and used in determining the positions of the points. A traverse may determine the relative positions of the points that it connects in a series.
- *Vertical angle*-An angle between two intersecting lines in a vertical plane. It should be understood that one line lies on the horizontal plane, and the angle originates from the intersection of the two planes.
- *Vertical line*-A line that lies in the vertical plane and is perpendicular to the plane of the horizon, such as the direction of a plumb line.
- *Vertical plane*-A plane that is perpendicular to the horizontal plane.

PART D - FIELDWORK

1-7. Fieldwork Concept. Fieldwork is important in all types of surveys. To be a skilled surveyor, you must spend a certain amount of time in the field to acquire needed experience. The study of this ACCP will enable you to understand the underlying theory of surveying, the instruments and their uses, and the surveying methods. However, a high degree of proficiency in actual surveying, as in other professions, depends largely on the duration, extent, and variety of your actual experience.

a. Project Analysis. The project must be analyzed thoroughly before going into the field. You must know exactly what is to be done, how you will do it, why you prefer a certain approach over other possible solutions, and what instruments and materials you will need to accomplish the project.

b. Speed. You must develop speed in all your fieldwork. This means that you will need practice in handling the instruments, taking observations, keeping field notes, and planning systematic moves. Surveying speed is not the result of hurrying; it is the result of the following:

- The surveyor's skill in handling the instruments.
- The intelligent planning and preparation of the work.

- The process of making only those measurements that are consistent with the accuracy level that is required.

c. Accuracy Levels. Measurements must not be accepted as correct without verification. Verification, as much as possible, must be different from the original measurement method used. The precision of measurement must be consistent with the accuracy level that is required for the surveying type being conducted. The higher the accuracy level, the more time that is required to make the measurement, since greater care and more observations must be taken.

The purpose and type of a survey are the primary factors in determining the accuracy level that is required. This, in turn, will influence the selection of instruments and procedures. First-order triangulation, which becomes the basis or "control" of future surveys, requires a high level of accuracy. At the other extreme, cuts and fills for a highway survey require a much lower accuracy level. In some construction surveys, inaccessible distances must be computed. The distance is computed by means of trigonometry, using the angles and the one distance that can be measured. The measurements must be made to a high degree of precision to maintain accuracy in the computed distance.

d. Maintenance. Fieldwork also includes adjusting instruments and caring for field equipment. Do not attempt to adjust any instrument unless you understand the workings or functions of its parts. Adjusting instruments in the early stage of your career requires close supervision from an experienced surveyor.

e. Factors Affecting Fieldwork. The surveyor must constantly be alert to the different conditions encountered in the field. Physical factors, such as terrain and weather conditions, will affect each field survey in varying degrees. The following are some of these conditions:

- Measurements using telescopes can be stopped by fog or mist.
- Swamps and floodplains under high water can impede taping surveys.
- Sighting over open water or fields of flat, unbroken terrain creates ambiguities in measurements using microwave equipment.
- Bright sunlight reduces light-wave measurements.

However, reconnaissance will generally predetermine these conditions and alert the surveying party to the best method to use and the rate of progress to be expected.

f. Training. The training status of personnel is another factor that affects fieldwork. Experience in handling the instruments being used for a survey can shorten surveying time without introducing errors that would require resurvey.

1-8. Measurements. Fieldwork in surveying consists mainly of taking and recording measurements. The operations are as follows:

- Measuring distances and angles for the purpose of-

- Establishing points and reference lines for locating details (such as boundary lines, roads, buildings, fences, rivers, bridges, and other existing features).
- Staking out or locating roads, buildings, landing strips, and other construction projects.
- Establishing lines parallel or at right angles to other lines.
- Extending straight lines beyond obstacles, such as buildings.
- Performing other duties that may require the use of geometric or trigonometric principles.
- Measuring differences in elevations and determining elevations for the purpose of-
 - Establishing reference points (bench marks).
 - Obtaining terrain elevations along a selected line for plotting profiles and computing grade lines.
 - Staking out grades, cuts, and fills for construction projects.
- Conducting topographic surveys where horizontal and vertical measurements are combined.
- Making soundings in bodies of water for preparing charts for navigation or for developing maps for waterfront structures.
- Recording field notes to provide a permanent record of the fieldwork.

Surveying measurements will be in error to the extent that no measurement is ever exact. Errors are classified as systematic and accidental. Besides errors, surveying measurements are subject to mistakes or blunders. These arise from misunderstanding of the problem, poor judgment, confusion on the part of the surveyor, or simply from an oversight. By working out a systematic procedure, the surveyor will often detect a mistake when some operation seems out of place. The procedure will be an advantage in setting up the equipment, making observations, recording field notes, and making computations.

1-9. Field Notes. The surveyor's field notes must contain a complete record of all measurements made during the survey, with sketches and narration when necessary, to clarify the notes. The best field survey is of little value if the notes are not complete and clear. They are the only record that is left after the field surveying party leaves the site.

a. **Field-Note Types.** The following are the four basic types of field notes: tabulations, sketches, descriptions, and combinations. The combination method is the most common method because it fits so many overall needs.

(1) Tabulations. Tabulations are the numerical measurements that are recorded in columns according to a prescribed plan. Spaces are also reserved to permit necessary computations. Tabulations, with or without added sketches, can also be supplemented with descriptions.

(2) Sketches. Sketches add much to clarify field notes and should be used liberally when applicable. They may be drawn to an approximate scale, or important details may be exaggerated for clarity. A small ruler or triangle is an aid in making sketches. Measurements should be added directly on the sketch or keyed in some way to the tabular data. A very important requirement of a sketch is legibility. See that the sketch is drawn clearly and large enough to be understandable.

(3) Descriptions. Descriptions may only be one or two words to clarify recorded measurements or may be a lengthy narration if it is to be used at some future time, possibly years later, to locate a surveying monument.

(4) Combinations. Two, or even all three, of the methods can be combined, when necessary, to make a complete record.

b. Field Notebook. A field notebook is a permanently bound book for recording measurements as they are made in the field. Several types are available to record the different kinds of surveying measurements. The front cover of a field notebook should be marked with the name of the project its general location, the types of measurements recorded, the designation of the surveying unit, and other pertinent information as specified by the engineering officer. The inside front cover should contain instructions for the return of the notebook, if lost. The right-hand pages should be reserved for an index of the field notes, a list of party personnel and their duties, a list of the instruments used (plus the dates and the reasons for any instrument being changed during the course of the survey), and a sketch and description of the project. Throughout the remainder of the notebook, the beginning and ending of each day's work should be clearly indicated. When pertinent, the weather, including temperature and wind velocities, should also be recorded. To minimize recording errors, all data entered in the notebook must be checked and initialed by someone other than the recorder.

(1) Legibility. All field notes should be lettered legibly. A mechanical pencil or a number 3 or 4 hard-lead pencil, using sufficient pressure, will ensure a permanent record. Numerals and decimal points should be legible and permit only one interpretation. Notes must be kept in the field notebook and not on scraps of paper for later transcription. Separate surveys should be recorded on separate pages or in different books.

(2) Erasures. Erasures are not permitted in field notebooks. Individual numbers or lines recorded incorrectly are lined out and the correct values added. Pages that are to be rejected are crossed out neatly and referenced to the substituted page. This procedure is mandatory since the field notebook is the book of record and is often used as legal evidence.

(3) Abbreviations. Standard abbreviations, signs, and symbols are used in field notebooks. If there is any doubt as to their meaning, an explanation must be given in the form of notes or legends.

PART E - SURVEYING PARTIES

1-10. Surveying-Party Types. The size of a field surveying party depends on the surveying requirements, the equipment available, the surveying method, and the number of personnel needed for performing the different functions. The following are the surveying parties that will be discussed in this lesson: a leveling party, a transit party, and a stadia party. The other surveying-party organizations generally follow the same pattern, therefore, they will not be discussed.

a. Leveling Party. The smallest leveling party consists of two persons. For differential leveling, one person acts as an instrument man (level man) and the other holds the rod (rod man). Trigonometric leveling requires instrument men to read vertical angles as well. In small parties of this type, the instrument men must record their own notes. Reciprocal leveling can be done by two people but requires separate vehicles for transporting the party and the equipment around the obstruction.

(1) Additional Persons. To improve the efficiency of the different leveling operations, additional personnel are required. A second rod man to alternate on the backsights and foresights will speed up leveling. A recorder will allow the instrument man to take readings as soon as the rod men are in position. For surveys with numerous side shots, extra rod men will eliminate waiting periods while one or two persons move from point to point. In surveys requiring a shaded instrument, an umbrella man can allow the recorder to concentrate on note keeping.

(2) Combined Party. Leveling operations may be run along with a traverse or as part of a taping survey. In these instances, the leveling party may be organized as part of a combined party with personnel assuming duties as required by the workload and as assigned by the party chief.

b. Transit Party. A transit party consists of at least three people: an instrument man, a head chainman, and a party chief. The party chief is usually the note keeper and may double as a rear chainman, or there may be an additional rear chainman. The instrument man operates the transit, the head chainman measures the horizontal distances, and the party chief directs the survey and keeps the notes.

c. Stadia Party. A stadia party should consist of three people: an instrument man, a note keeper, and a rod man. However, two rod men should be used if there are long distances between observed points so that one can proceed to a new point while the other is holding the rod on a point being observed. The note keeper records the data the instrument man calls out and makes the required sketches.

PART F - SURVEYS

1-11. Surveying Types. Generally, surveys are classified by names descriptive of their functions, such as property surveys, mine surveys, hydrographic surveys, and so on. Although surveys are classified by many different names, the methods and instruments used are basically the same. Some of the types of surveys that you might perform as a construction surveyor are discussed below.

a. Land Surveys. A land survey is conducted to establish the exact location, boundaries, or subdivision of a tract of land in a specified area. This type of work is sometimes referred to as cadastral surveying. When this work is primarily done within city limits, it is referred to as city surveying. At present, land surveys generally consist of the following chores:

- Establishing markers or monuments to define and thereby preserve the boundaries of land belonging to an individual, a corporation, or the government.
- Relocating markers or monuments legally established by original surveys. This requires examining previous surveying records and retracing what was done. When some markers or monuments are missing, they are reestablished by following recognized procedures and using whatever information is available.
- Rerunning old land-surveying lines to determine their lengths and directions. As a result of the high cost of land, old lines are remeasured to get more precise measurements.
- Subdividing land into parcels of predetermined sizes and shape.
- Calculating areas, distances, and directions and preparing a land map to portray surveying data so that it can be used as a permanent record.
- Writing a technical description for deeds.

b. Topographic Surveys. A topographic survey is conducted to gather surveying data about natural and man-made land features, as well as elevations. From this information a three-dimensional map may be prepared. The topographic map may be prepared in the office after collecting the field data or done right away in the field by plane table. The work usually consists of the following:

- Establishing horizontal and vertical control that will serve as the framework of the survey.
- Determining the horizontal location and elevation (usually called "side shots") of ground points to provide enough data for plotting when the map is prepared.
- Locating natural and man-made features.
- Computing distances, angles, and elevations.
- Drawing the topographic map.

c. Engineering or Construction Surveys. An engineering or a construction survey is conducted to obtain data for the various phases of construction activity. It includes a

reconnaissance survey, a preliminary survey, a location survey, and a layout survey. The objectives of an engineering or a construction survey include-

- Obtaining reconnaissance information and preliminary data that engineers require for selecting suitable routes and sites and for preparing structural designs.
- Defining selected locations by establishing a system of reference points.
- Guiding construction forces by setting stakes or marking lines, grades, and principal points and by giving technical assistance.
- Measuring construction items in place to prepare progress reports.
- Dimensioning structures for preparing as-built plans.

(1) Terms. The American Society of Civil Engineers (ASCE) applies the term engineering surveys to all of the above objectives and construction surveys to the last three objectives only. The Army Corps of Engineers, on the other hand, applies construction surveys to all of the above objectives.

(2) Structures. Engineering and/or construction surveys form part of a series of activities leading to the construction of a man-made structure. The term "structure" is usually confined to something that is built of structural members, such as a building or a bridge. It is used here in a broader sense, however, to include all man-made features, such as graded areas; sewer, power, and water lines; roads and highways; and waterfront structures.

d. Route Surveys. A route survey is conducted for locating and constructing transportation or communication lines that continue across country for some distance, such as highways, railroads, open conduit systems, pipelines, and power lines. Generally, the preliminary survey for this type of work takes the form of a topographic survey. In the final stage, the work may consist of the following:

- Locating a centerline, usually marked by stakes at 100-foot intervals (called stations).
- Determining elevations along and across a centerline for plotting a profile and cross sections.
- Plotting a profile and cross sections and fixing grades.
- Computing the volumes of earthwork and preparing a mass diagram.
- Staking out the extremities for cuts and fills.
- Determining drainage areas to be used for ditches and culverts.
- Laying out structures, such as bridges, culverts, and so on.

- Locating right-of-way boundaries, as well as staking out fence lines, if necessary.

PART G - CONTROL SURVEYS

1-12. Control Types. Control surveys establish reference points and reference lines for detail surveys. Control may be either horizontal or vertical.

a. Horizontal Control. Horizontal control is a basic framework of points in which the horizontal position and interrelationship of have been accurately determined.

(1) Horizontal Control by Traversing. A surveying traverse is a sequence of lengths and directions of lines between points on the earth, obtained by or from field measurements and used in determining the positions of the points. A surveying traverse may determine the relative positions of the points that it connects in a series.

(a) Closed Traverse. A closed traverse is one that ends at the point at which it began (see Figure 1-4, page 1-16).

(b) Open Traverse. An open, or open-end, traverse is one that ends at a point other than the one at which it began (see Figure 1-5, page 1-16).

(2) Horizontal Control by Triangulation. Triangulation is a method of surveying in which the stations are points on the ground that are located in a series of triangles. The angles of the triangulation net are measured by using instruments, and the lengths of the sides are derived by computation from selected sides that are termed baselines—the lengths of which have been obtained from precise direct measurements on the ground.

b. Vertical Control. Vertical control (also called elevation control) is a series of bench marks or other points of known relative vertical position that are established throughout a project. In a topographic survey, for example, a circuit of bench marks is established over an area at convenient intervals (usually every half mile along a coordinate system on government property) to serve as starting and closing points for leveling operations. They also serve as reference marks for grades and finished floor elevations for structures in subsequent construction work. Since these bench marks will be needed from time to time to establish other elevations, it is important that the work be accurately done so that elevations referred to by one bench mark will check with those referred to by any other bench mark in the circuit. The bench marks must be established in a definite point of more or less permanent character so that they will not be disturbed.

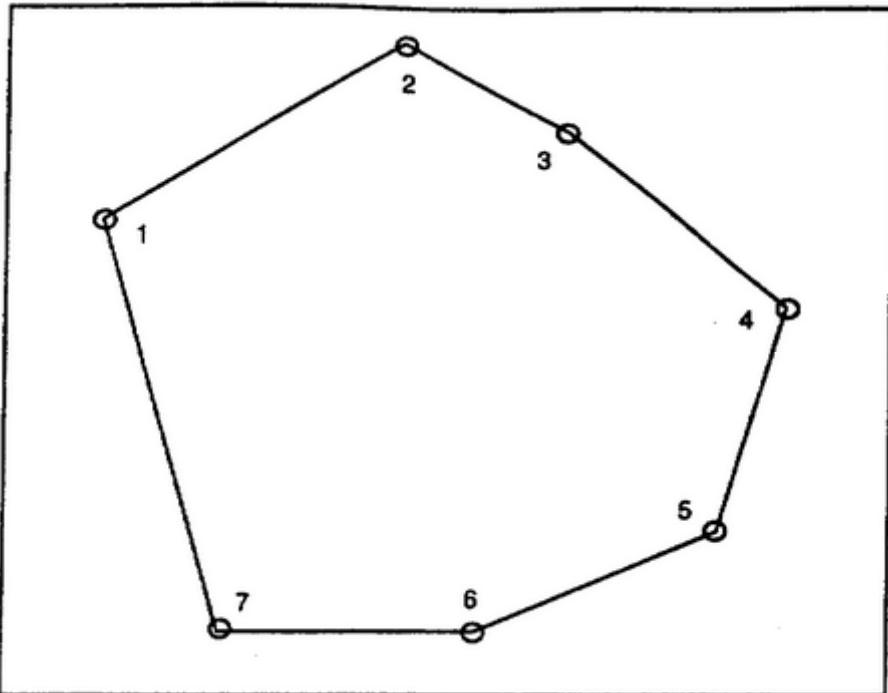


Figure 1-4. Closed traverse

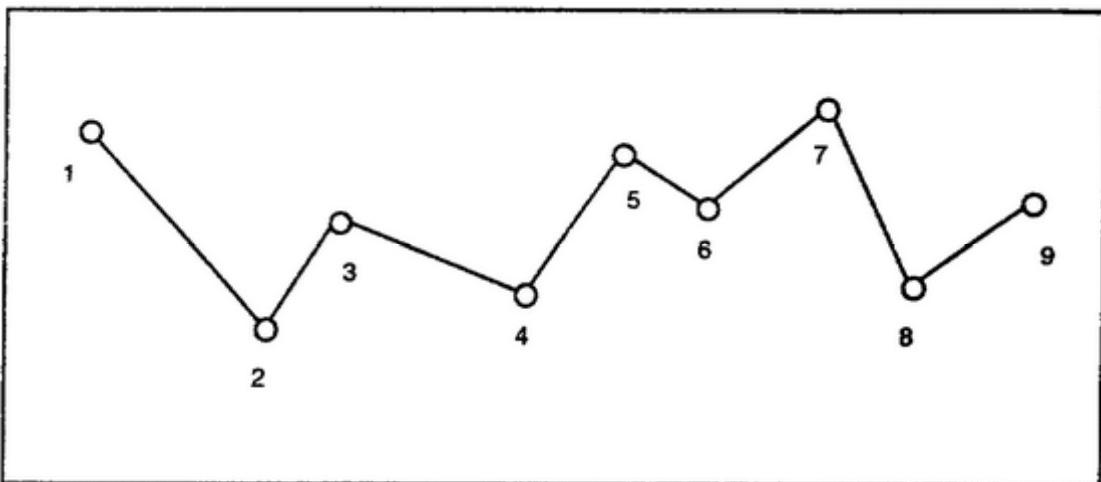


Figure 1-5. Open traverse

PART H - PROFESSIONAL SOCIETIES AND MANUFACTURERS

1-13. Surveyors' Professional Societies. US surveyors have two professional societies: the Surveying and Mapping Division of the ASCE and the American Congress on Surveying and Mapping. You must be a registered civil engineer to become a member of the ASCE or a registered surveyor to become a member of the American Congress on Surveying and Mapping. A working committee of these organizations may be formed to resolve technical problems affecting the art and science of surveying, such as the

Committee of Surveying and Mapping Division, ASCE. The executive committee of these organizations may also appoint representatives to represent them in international conventions, such as the International Geodetic and Geophysical Union.

1-14. Manufacturers of Surveying Instruments and Supplies. Berger, Bruning, Dietzgen, Gurley, Kern, Keuffel & Esser, Lufkin, Post, Litton Systems Inc. and Wild, along with others, are well-known manufacturers of surveying equipment. Manufacturers publish, in pamphlets and booklets of various kinds, a great deal of valuable information on surveying equipment and its use and are usually glad to provide such information, without charge, to anyone requesting it.

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LESSON 1

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. The location of a point may be determined by its _____ from another point.
 - A. Elevation
 - B. Distance and direction
 - C. Horizontal angle
 - D. Vertical angle

2. What determines the accuracy level that is required for a survey?
 - A. Time
 - B. Weather
 - C. Equipment
 - D. Purpose

3. Measurement errors are classified as _____.
 - A. Systematic and accidental
 - B. Continuous and retracted
 - C. Horizontal and vertical
 - D. Accidental and deliberate

4. What type of survey is used to gather data on natural and man-made land features?
 - A. Construction
 - B. Land
 - C. Topographic
 - D. Route

5. Elevation control is another term for _____.
 - A. Elevation
 - B. Horizontal distance
 - C. Angle of elevation
 - D. Vertical control

LESSON 1

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct Answer and Feedback
1.	B Distance and direction A point may also be...(page 1-3, para 1-4)
2.	D Purpose The purpose and type...(page 1-9, para 1-7c)
3.	A Systematic and accidental Errors are classified as...(page 1-10, para 1-8)
4.	C Topographic A topographic survey is conducted...(page 1-13, para 1-11b)
5.	D Vertical control Vertical control (also called elevation control)...(page 1-15, para 1-12b)

LESSON 2

RATIOS, PROPORTIONS, ROOTS, AND POWERS

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use mathematics.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use ratios, proportions, roots, and powers.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Surveying operations require that you have a thorough understanding of mathematics to complete even the simplest surveying task. The fact that a calculator may accompany you on a survey mission does not excuse you from applying your knowledge of mathematics. As a surveyor, you need to know mathematics and must be able to make all the necessary computations in the following broad areas:

- Doing various kinds of field computations that are required to determine accurate lengths, geographic positions, and horizontal and vertical angles, as well as curves, grade lines, elevations, and earthwork volumes.
- Reviewing and checking all field data and values for completeness and accuracy.

In addition to the simple elements of mathematics, higher or more difficult forms of "math" are needed in surveying operations. In this lesson, you will refresh your understanding of ratios, proportions, roots, and powers. In the following lessons, you will study the more advanced branches of mathematics, such as geometry and trigonometry.

PART A - RATIOS AND PROPORTIONS

2-1. Defining Ratios and Proportions. You will find that a ratio can be expressed in four different ways. For example, the side-slope ratio of 2-to-1 is normally used with soft clay. This ratio can be expressed as follows: 2-to-1, 2:1, $\frac{2}{1}$, or $2\frac{1}{1}$. The numbers 2 and 1, which are terms of the ratio, are called the antecedent and the consequent, respectively. The antecedent is the same as the dividend or numerator, and the consequent is the same as the divisor or denominator. Both terms of the ratio can be multiplied or divided by the same number without changing the value of the ratio. In the ratio $12/3$, for example, the number 12 is divided by 3, giving the value of 4. This means that the ratio $12:3$ is equal to the ratio $4:1$. Other examples are shown below.

Example 1: What is the ratio of 6:2? Set up the problem as a fraction, and perform the indicated operation.

Solution: $6/2 = 3$, or $3S1$

Example 2: What is the ratio of 7:3? Set up the problem as a fraction, and perform the indicated operation.

Solution: $7/3 = 2\frac{1}{3}$, or $2\frac{1}{3}:1$

a. A proportion is a statement of equality between two ratios. If the value of one ratio is equal to the value of another ratio, they are said to be a proportion. For example, the ratio 3:6 is equal to the ratio 4:8. Therefore, this relationship can be written in one of the following forms:

(1) $3:6 :: 4:8$

(2) $3:6 = 4:8$

(3) $\frac{3}{6} = \frac{4}{8}$

b. In any proportion, the first and last terms are known as the extremes and the second and the third terms are known as the means. If you look at the proportion example in the previous paragraph, you note that the terms "3" and "8" are the extremes, while the terms "6" and "4" are the means.

c. When working proportions, you should remember that there are three rules which are used in determining an unknown quantity. These rules can also be used to prove that the proportion is true.

(1) The first rule is that in any proportion the means' product equals the extremes' product. See the following examples:

(a) $2:3 :: 6:9$

(b) $2 \times 9 = 18$ (extremes' product)

$$(c) \quad 3 \times 6 = 18 \text{ (means' product)}$$

When the same problem is expressed in another form, the operation remains the same.

$$\frac{2}{3} = \frac{6}{9}$$

In this case, the proportion is expressed in a fractional form so that the numerator of one fraction is multiplied by the denominator of the other fraction. This is called the cross-multiplication process.

$$2 \times 9 = 18 \text{ (extremes' product)}$$

$$3 \times 6 = 18 \text{ (means' product)}$$

(2) The second rule is that in any proportion the means' product divided by either extreme gives the other extreme.

$$3:4 :: 6:8$$

$$4 \times 6 = 24 \text{ (means' product)}$$

$$\frac{24}{8} = 3 \text{ (other extreme)}$$

(3) The third rule is that in any proportion the extremes' product divided by either mean will determine the other mean.

$$2:3 :: 4:6$$

$$2 \times 6 = 12 \text{ (extremes' product)}$$

$$12/3 = 4 \text{ (one mean)}$$

$$12/4 = 3 \text{ (other mean)}$$

d. The basic knowledge and appropriate use of the three proportion rules will aid you in determining the value of an unknown term when the other three terms are known. A typical use of proportion involves using the representative-fraction (RF) formula ($RF = MD/GD$), which is effective in determining ground distance (GD), map distance (MD), or the RF. Most, if not all, maps express their scales as a ratio or RF; that is, one unit on the map is equal to so many units on the ground. This map-to-ground relationship is extremely versatile since any kind of unit can be used to determine the map's RF. Thus, 1 inch on the map is equal to so many inches on the ground; the same ratio holds true for feet, meters, or yards. As a result, a map of a certain scale, or RF, may show that 1 foot on the map is equal to 20,000 feet on the ground. Therefore, the map's RF is 1:20,000. If a map of this scale were to be used, it would be easy to determine the actual distance (or linear measurement) of a survey line on the ground. See the following example:

Example: What is the length of a survey line on the ground if it measures 5 inches on a map having a RF of 1:20,000? Set up the problem using the RF formula ($RF = MD/GD$). Use the known information and write the proportion. Let x represent your unknown extreme.

$$RF = 1:20,000$$

$$MD = 5 \text{ inches}$$

$$x = GD$$

Solution: $1:20,000 :: 5:x$

Using the rule of the means' product divided by the known extreme (1), find the unknown extreme (x), which, in this case, represents the GD.

$$(5 \times 20,000)/1 = 100,000$$

The answer is given in inches, which should be converted to the more conventional surveyor's units: yards, feet, meters, or miles. This requires that you divide the answer by 36, 12, 39.37, or 63,360, respectively.

e. The use of ratios and proportions can also serve you in other situations. See the following example:

Example: If a survey line measures 7,920 feet on the ground, what is the length of it on a map having a RF of 1:31,680? Set up your proportion using the RF formula ($RF = MD/GD$). Since the MD is usually expressed in inches, change the GD to inches by multiplying by 12. Let x equal the unknown mean.

$$RF = 1:31,680 \text{ (scale of progress map)}$$

$$GD = 7,920 \text{ feet (length of surveyed line)}$$

$$x = MD$$

Solution:

$$1:31,680 :: x:7,920 \cdot 12$$

$$1:31,680 :: x:95,040$$

Using the rule of the extremes' product divided by the known mean (31,680), find the unknown mean (x), which, in this case, represents the desired MD.

$$(1 \times 95,040) / 31,680 = 3 \text{ inches}$$

f. Up to this point, only direct proportion has been discussed. In a direct proportion, both ratios are direct ratios; that is, they increase or decrease in the same manner. In the map proportion problem above, the first ratio (1:31,680) increased in the same manner as the second ratio (3:95,040). This problem would also be a direct proportion if the terms had decreased in a like manner. Most of the proportion problems you will use are direct proportions and, unless specifically noted, will be treated as such. A simple clue that will aid you is to analyze each problem carefully to determine whether the unknown term will be greater or lesser than the known term of the ratio in which it occurs. Thus, each direct proportion takes the following pattern:

Therefore, when setting up a direct proportion problem, make sure that the ratio is stated correctly. Failure to do so may result in another type of proportion known as an inverse proportion, which will give you the wrong answer.

2-2. Defining Inverse Proportions. The term "inverse proportion" is given to those problems where an increase in the value of one term will cause a decrease in the value of another term. The same would hold true if a decrease in one term would cause an increase in another term. For example, the ratio 3:2 is the inverse of the ratio 2:3, thus when the two ratios are equated, the terms, or elements, are said to be inversely proportional. In this example, the product of each ratio is equal.

3:2 :: 2:3 or 2:3 :: 3:2

a. Another example is where the means of a proportion can be changed in an inverse manner while the extremes are held constant (the same can be done with the extremes and the means held constant). For example, in the proportion $2:5 :: 8:20$, the means' product (5×8) is 40. If the first mean is doubled and the second mean is halved, the proportion becomes $2:10 :: 4:20$. The means' product is still 40 even though the means were inversely changed. You will note that the means' product of both proportions is equal to the extremes' product; thus, the proportional relationship between the ratios remains the same. A similar situation exists if the extremes are changed.

2:5 :: 8:20

Doubling the first extreme and halving the remaining extreme changes the proportion but not the relationship between ratios.

4·5 .. 8·10

Any of the three proportion rules may be applied to the examples above to determine an unknown value and to prove that a proportion is true.

b. Proportion problems can be used effectively in determining the manpower needed on a proposed survey project. For example, it is necessary to estimate the number of days needed to survey a pipeline right-of-way in a tropical country. Although the line is long, this assignment normally takes 252 days for 2 survey teams to complete. However, adverse weather conditions are expected in about 60 days; therefore, can 12 survey teams complete the assignment before day 60?

c. Your analysis should indicate that if the number of survey teams increases, the number of days to complete the assignment will decrease. Therefore, in order to solve this problem, use an inverse proportion, whereby one term decreases as another term increases. The best way to set up an inverse proportion is to equate like terms, such as 2 teams:12 teams and 252 days:x days. You infer that the time involved with 12 survey teams is less than that required with 2 teams therefore, you can invert the

time ratio within the proportion and carry out its solution.

$$2:12 :: x:252$$

d. By using the rule of the extremes' product divided by the known mean to determine the unknown mean, you will find that 12 survey teams can accomplish the task in 42 days--well before day 60.

PART B - ROOTS AND POWERS

2-3. Defining Roots and Powers. If you were asked to multiply 842 by itself and 369 by itself, it would mean that both of these numbers were raised to their second powers. Actually, a number can be raised to any power, and this fact is indicated by an exponent. The exponent is a number placed somewhat above and to the right of the number. Thus, 842×842 can be written in its proper form of 842^2 . Other powers of a number are indicated by the appropriate exponent. For example, $5 \times 5 \times 5 \times 5 \times 5$, which is written 5^5 , means that the number 5 must be raised to its fifth power. Thus, the power indicates a special case of multiplication and enables you to express numbers in a form that is useful for special application.

a. If the power is a low-value integer, it can be readily computed by arithmetic methods. Powers that are large numbers or decimals are usually determined through the use of logarithm tables or, if available, computers. Low-value exponents are frequently in many of the field-survey computations, particularly where known values are substituted for lettered terms that often must be squared (raised to the second power). One of the formulas that a surveyor frequently uses while in the field is $a^2 + b^2 = c^2$; each letter represents a side of a right triangle (one that contains a 90° angle). Generally, a surveyor can lay out a right triangle and measure two sides where some obstacle or obstruction prevents measuring the third side. Thus, if sides a and b are known, it is simple to find the value of c.

b. Equal factors of a number are known as roots; therefore, finding the root of a number is the reverse of finding the power. When two equal factors are found for a number, each is known as the second root, or square root. For example, the square root of 25 is 5, since 5×5 or $5^2 = 25$. When three equal factors are found, each factor is known as a third root, or cube root. The cube root of 64 is 4, since $4 \times 4 \times 4$ or $4^3 = 64$. Four equal factors are known as a fourth root, five equal factors are known as a fifth root, and so on.. Square and cube roots can readily be determined by an arithmetic method. However, only square roots are normally found by this method, since cube-root determinations are very involved and time consuming.

c. In determining the root of a number, you must use these two notations; the radical sign and the exponent. The radical sign ($\sqrt{}$) is combined with the vinculum ($\overline{}$) and placed in front of a number to indicate that an extraction of its square root is necessary. When a root other than a square root needs to be determined, this is indicated by a small index number placed in the angle formed by the radical sign as shown below.

$$\sqrt[3]{216} = 6$$

$$\sqrt[4]{81} = 3$$

$$\sqrt[5]{16807} = 7$$

d. Often, you will find that the exponent is used to show the particular root extraction of a number that is desired. For example, the square-root extraction of a number can be indicated by the exponent 1/2, the cube root by 1/3, the fourth root by 1/4, the fifth root by 1/5, and so forth. Examples of the equal factor or root determination by exponents is shown below. In each example, the root 3 was needed.

$$9^2 (9 = 3 \times 3)$$

$$27^{1/3} (27 = 3 \times 3 \times 3)$$

$$81^{1/4} (81 = 3 \times 3 \times 3 \times 3)$$

$$243^{1/5} (243 = 3 \times 3 \times 3 \times 3 \times 3)$$

e. According to your proficiency of common multiplication, you can determine mentally the square root of some numbers. For example, the square root of $25^{1/2}$ is 5, since 5×5 or $5^2 = 25$. Similarly, the square root of $144^{1/2}$ is 12, since 12×12 or $12^2 = 144$. In other cases, however, the square root of a number must be determined by a mathematical process. If the number is a perfect square, the square root will be an integral number; if the number is not a perfect square, the square root will be a continued decimal.

f. Currently, there are tables that are designed to determine the root of any number. However, if these tables are unavailable to you while in the field, you must be able to determine the square root of any number. For example, suppose that you have just laid out one leg of a triangle and want to calculate the square root. The length of the leg (or side) is 3,398.89 feet. The first step would be to separate the number (3,398.89) into 2-digit groups, starting from the decimal point and working in both directions.

$$\sqrt{3398.89}$$

g. Next, you will place the decimal point for the intended answer directly above the decimal point that appears in the dividend under the radical sign. The square root for this number will have 1 digit for each 2-digit group. Now determine the largest number that can be squared without exceeding the first pair of digits (33). The answer is 5, since the square of any number larger than 5 will be greater than 33. Place the digit 5 above the first pair of digits in the dividend. Squaring 5, place the product under the first two digits (33), and perform the indicated subtraction. Now bring down the next pair of digits as shown below.

$$\begin{array}{r} 5 \\ \sqrt{3398.89} \\ 25 \\ \hline 898 \quad (\text{new dividend}) \end{array}$$

h. To obtain the next trial divisor, the digit 5 is doubled and placed to the left of the new dividend. Divide the trial divisor (10) into all but the last digit of the new dividend; it will go into 89

eight times. Place the number 8 above the second pair of digits, and also place the number 8 to the right of the trial divisor. Thus, the new trial divisor is now 108. Multiply the new trial divisor by 8, and write the product just under the former remainder (898). Perform the indicated subtraction, and bring down the next pair of digits (89).

$$\begin{array}{r}
 5 8. \\
 \sqrt{3398.89} \\
 \underline{25} \\
 898 \\
 \underline{864} \\
 34 89 \quad (\text{new dividend})
 \end{array}$$

$5 \times 2 = 10(8)$

$8 \times 108 =$

i. To obtain the next trial divisor, double the partial answer (58). Divide the trial divisor (116) into all but the last digit of the new dividend; it will go into 348 three times. Place the digit 3 to the right of the decimal point in the quotient immediately above the third pair of digits in the original dividend. Also, place the 3 to the right of the trial divisor. Thus, the true divisor is 1163. Multiply 1163 by 3 to obtain a product of 3,489, which is written under the new dividend. Since there is no remainder, you may consider the figure 3,398.89 as a perfect square, and its square root is 58.3.

$$\begin{array}{r}
 5 8. 3 \\
 \sqrt{3398.89} \\
 \underline{25} \\
 898 \\
 \underline{864} \\
 2 \times 58 = 116(3) \quad 34 89 \\
 3 \times 1163 = 34 89 \quad (\text{new dividend})
 \end{array}$$

j. The final step in the square-root-extraction process should be a check of your work. This check can be done by squaring 58.3 and arriving at the dividend shown under the radical sign.

LESSON 2

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part of the lesson which contains the portion involved.

1. $2^7 = \underline{\hspace{2cm}}$.
A. 7×7
B. $2 \times 2 \times 2 \times 2 \times 2 \times 2$
C. 14
D. 128
2. The number 6 in the expression 3^6 is called the _____.
A. Square
B. Exponent
C. Cube
D. Proportion
3. What is the ratio of 5-to-3?
A. 15-to-6
B. 3-to-1
C. .06-to-1
D. 1.67-to-1
4. The expression 6-to-9 = 5-to-8 is referred to as being _____.
A. A process
B. A ratio
C. A proportion
D. invalid
5. What is the value of the missing extreme in the expression $2:12,000 = 3:\underline{\hspace{2cm}}?$
A. 18,000
B. 6,000
C. 24,000
D. Cannot be determined

LESSON 2

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

<u>Item</u>	<u>Correct answer and feedback</u>	
1.	D	128 Equal factors of a number are known...(page 2-6, para 2-3b)
2.	B	Exponent The exponent is a number...(page 2-6, para 2-3)
3.	D	1.67-to-1 You will find that a ratio can be expressed...(page 2-2, para 2-1)
4.	C	Proportion If the value of one ratio is equal...(page 2-2, para 2-1a)
5.	A	18,000 The first rule is that in any proportion...(page 2-2, para 2-1c(2))

LESSON 3

GEOMETRY

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use geometry.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will learn how to use geometry for surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Geometry deals with lines, angles, surfaces, and solids. In general, the lengths of lines or the sizes of angles (measurable in degrees or parts of a degree) are not measured by divisions on a ruler but are compared with one another. One of the most important uses of geometry is understanding other branches of math, which are needed to solve problems relating to geodetic positions, elevations, areas, or volumes of earthwork or other material. Also, understanding geometry helps in understanding trigonometric principles. A surveyor will use geometry in his daily work.

PART A - LINES

3-1. General Information. A line is generated by a point in motion; it has the dimension of length but not thickness. Dots and lines made on a drawing actually have thickness and are merely convenient representatives of points and lines. A line that has the same direction for its entire length is called a straight line. A line that changes in direction along its length is called a curved line. A flat surface is generated by a straight line moving in a direction other than its length. A surface has the dimensions of length and width but not thickness.

PART B - ANGLES

3-2. General Information. An angle can be identified by the symbol for its vertex. In Figure 3-1, the vertex is called angle O. This method can be used if there are no other angles at point O to cause confusion. However, in Figure 3-2, four angles have their vertex at point O, therefore, this method cannot be used. The letters of the sides and vertex are commonly used in geometry to identify angles. This method is more exact and should leave no question as to which angle is referenced. In Figure 3-1, the angle can be identified as AOB. The symbol \angle can be used in place of the word "angle." Thus, angle AOB can be written as $\angle AOB$ or $\angle O$. A symbol, such as the Greek letter θ , can be used to identify an angle. This method is generally used in trigonometry.

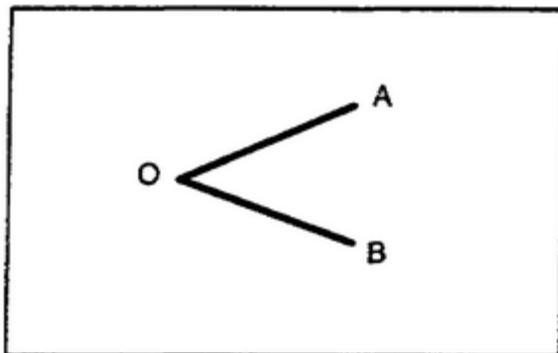


Figure 3-1. Angle O

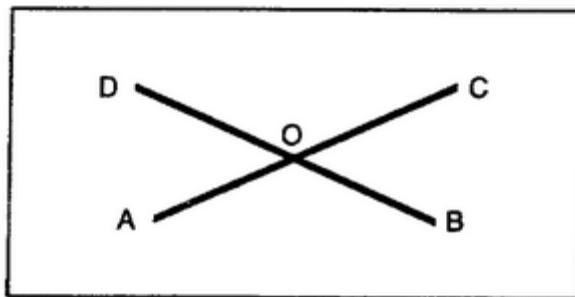


Figure 3-2. Angle AOB

Figure 3-3 shows the five general classes of angles. An angle that-

- Is formed by perpendicular lines is a right angle.
- Is less than a right angle is an acute angle.
- Is greater than a right angle but less than a straight angle is called an obtuse angle.
- Has sides which extend in opposite directions from the vertex is called a straight angle.
- Is greater than a straight angle but less than two straight angles is called a reflex angle.

Angles can also be classified into two general groups: right and oblique. Oblique angles are all angles other than straight and right angles.

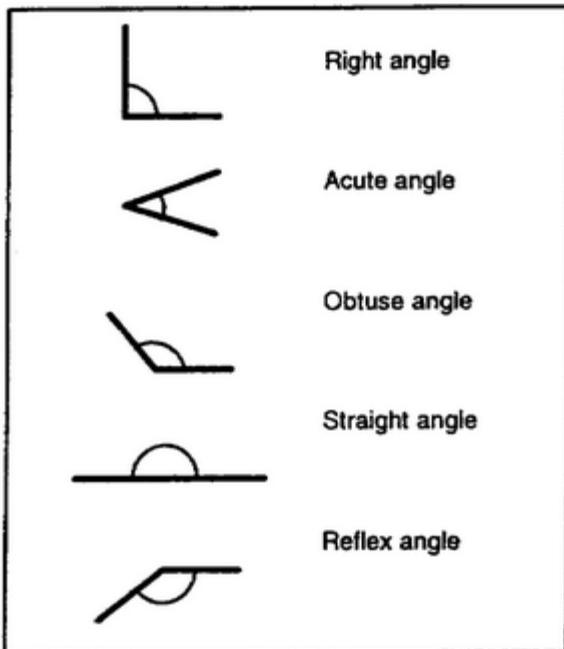


Figure 3-3. Classes of angles

a. Adjacent Angles. Angles that have a common vertex and a common side between them are called adjacent angles. Thus, in Figure 3-2, BOC and COD are adjacent angles; however, BOC and AOD are not adjacent angles because they have no common side. When one straight line meets another straight line to form two equal adjacent angles, the lines are perpendicular to each other, and the angles are right angles. In Figure 3-4, line CO is perpendicular to line AB, and angles BOC and AOC are right angles. The small square at the point where the lines meet is used to indicate that a right angle is formed by the two lines.

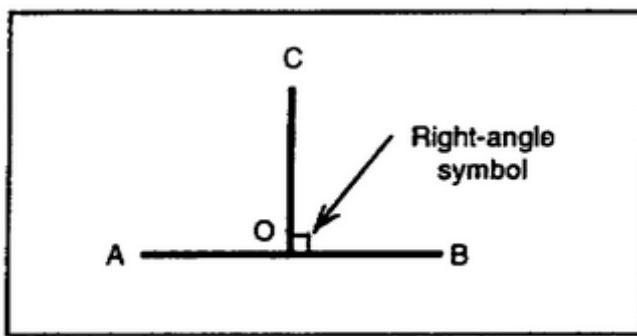


Figure 3-4. Right angles

b. Related Angles. General relationships between angles are shown in Figure 3-5, page 3-4. When two combined angles are equal to a right angle, they are called complementary angles. In Figure 3-5, angle AOC is a right angle, and angles AOB and BOC are complementary angles. Angle AOB is the complement of BOC, and angle BOC is the complement of AOB. When two combined angles are

equal to a straight angle, they are called supplementary angles. In Figure 3-6, angle AOB is a straight angle, and angles AOC and BOC are supplementary angles. Angle AOC is the supplement of BOC, and angle BOC is the supplement of AOC. When two lines intersect, the opposite angles are called vertical angles. In Figure 3-7, angles AOD and BOC are vertical angles, and angles AOC and BOD are also vertical angles.

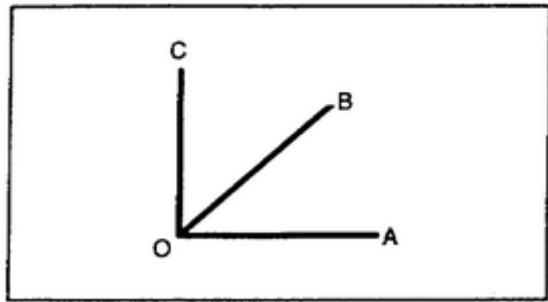


Figure 3-5. Complimentary angles

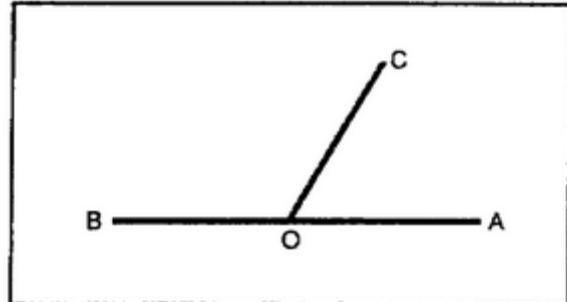


Figure 3-6. Supplementary angles

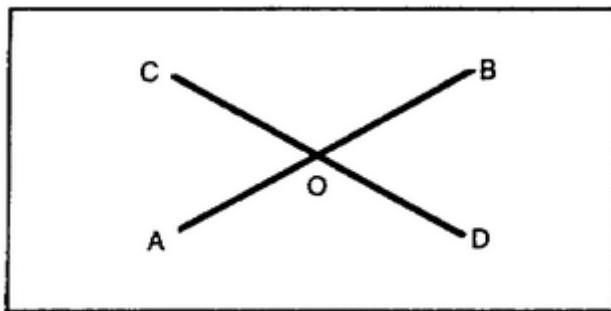


Figure 3-7. Vertical angles

3-3. Measuring Angles. Measuring angles is important in calculations for surveying problems. Degrees ($^{\circ}$) are used for angle measurements. An angular degree is $1/360$ th of a circle. The degree is further divided into minutes ($'$) and seconds ($''$), with 60 minutes in 1° and 60 seconds in 1 minute.

3-4. Generating Angles. You can generate an angle by rotating a line as shown in Figure 3-8. Rotate line OP in a counterclockwise direction about point O, starting from line AO. By this action the angle AOP is generated. When line OP is at one-quarter of a revolution, it is perpendicular to line AO, and a right angle is formed. When you rotate line OP completely around the circle, it generates four right angles (360°). Thus, the total angular magnitude about a point in a place is equal to four right angles.

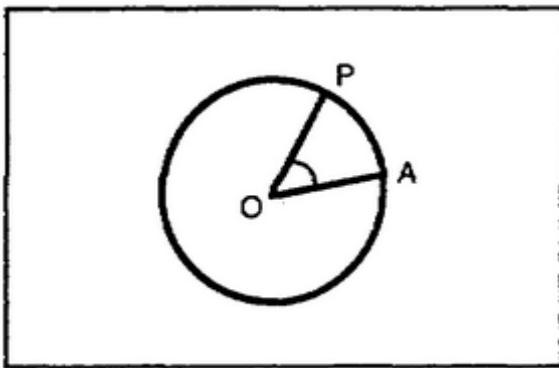


Figure 3-8. Generating an angle

PART C - TRIANGLES

3-5. General Information. A portion of a plane that is bounded by three straight lines is a plane triangle, or simply a triangle. The lines that bound the triangle are called its sides, and the sum of their lengths is its perimeter.

a. An angle that is formed within a triangle by any two sides is called an interior angle. An angle that is formed outside a triangle by any side and the extension of another side is called an exterior angle. Interior and exterior angles of a triangle are shown in Figure 3-9. Whenever the angles of a triangle are mentioned, the interior angles are referenced unless otherwise specified. The interior angles that are not adjacent to a given exterior angle are called opposite interior angles.

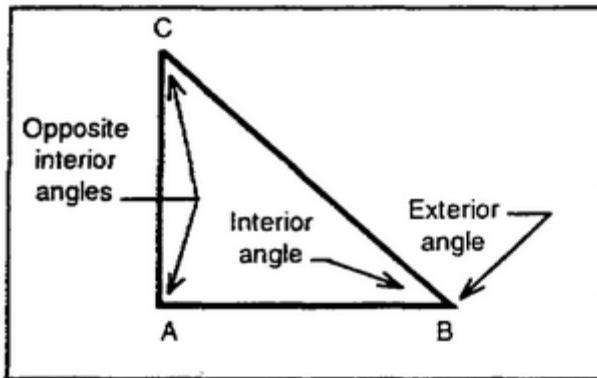


Figure 3-9. Angles

b. One side of a triangle is usually drawn as a horizontal line with the other two sides above it. The bottom side is then called the base of the triangle, as shown in Figure 3-10, page 3-6. However, any side of a triangle can be designated as its base, and it is not necessary to draw it at the bottom of the figure. The angle opposite the base is called the vertex angle, and the vertex of this angle is called the vertex of the triangle. The perpendicular distance from the vertex of a triangle to its base, or to an extension of its base, is called the altitude of the triangle. The side of a triangle that is opposite a right angle is called the hypotenuse.

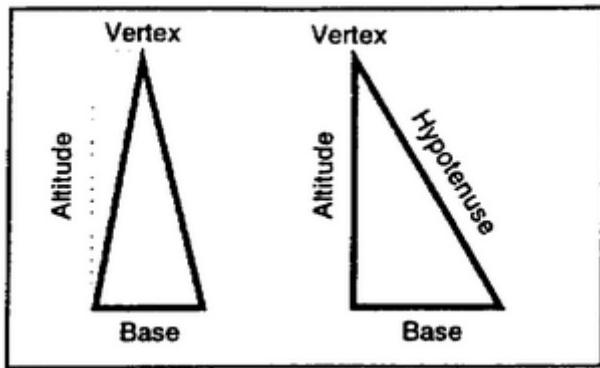


Figure 3-10. Sides and altitude

c. The general types of triangles are shown in Figure 3-11. They are as follows:

- (1) An acute triangle has interior angles that are less than right angles.
- (2) A right triangle has one right angle and two acute angles.
- (3) An obtuse triangle has one obtuse angle and two acute angles.
- (4) An isosceles triangle has two equal sides and two equal angles.

(5) An equilateral triangle has three equal sides and three equal angles and is sometimes called an equiangular triangle. All of the angles within an equilateral triangle are less than right angles; therefore the equilateral triangle is also an acute triangle.

d. Obtuse and acute triangles are often called oblique triangles to distinguish them from right triangles. Triangles with three unequal sides are called scalene triangles.

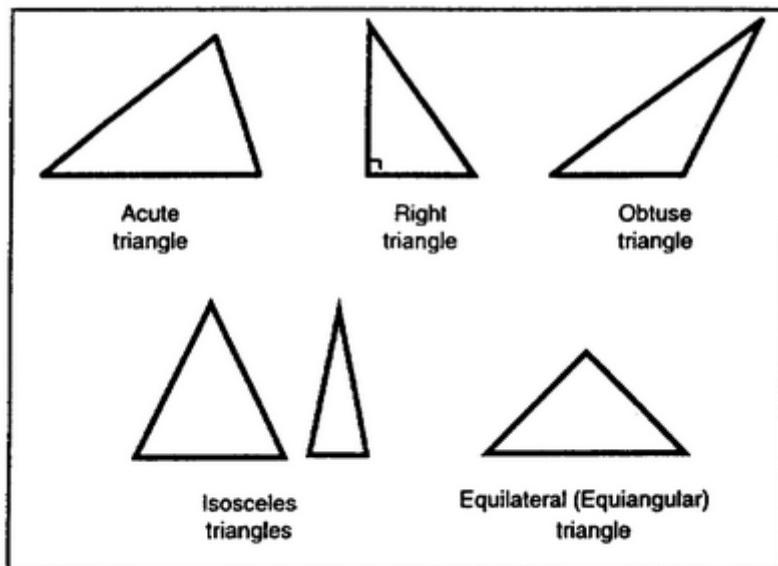


Figure 3-11. Types of triangles

PART D - POSTULATES

3-6. Basic Concepts of Postulates. Fundamental principles of geometry that depend on personal observation and must be taken without proof are called postulates. Postulates are used to develop rules for other branches of mathematics and to prove many arithmetic and algebraic relationships. The basic postulates are that-

- a. A straight line is the shortest distance between two points.
- b. Only one straight line can be drawn between the same two points.
- c. Two straight lines cannot intersect at more than one point.
- d. A straight line may be extended indefinitely.
- e. A straight line may be drawn from a point to any other point.
- f. A geometric figure may be moved from one position to another without any change in form or magnitude.
- g. All right angles (90° angles) are equal.
- h. All straight angles (180° angles) are equal.
- i. Only one perpendicular line can be drawn from a point in a line in a plane.
- j. Only one perpendicular line can be drawn from a point outside a line in a plane.
- k. Two adjacent angles which have their exterior sides in a straight line are called supplementary angles.
 - l. The sum of all angles about a point on one side of a straight line in a plane is equal to two right angles (180°).
 - m. The sum of all angles about a point in a plane is equal to four right angles (360°).
 - n. Angles that have the same supplement are equal, and angles that have the same complement are equal.
 - o. Vertical angles are equal.
 - p. A line segment can be bisected in only one point.
 - q. An angle can be bisected by only one line.
 - r. The sides of a square are equal.

- s. Only one circle can be drawn with a given point as center and a given distance as radius in a plane.
- t. All radii of the same circle or of equal circles are equal.
- u. All diameters of the same circle or of equal circles are equal.
- v. A straight line can intersect a circle at only two points. If the two points coincide, the straight line is tangent to the circle.
- w. A circle can intersect another circle at only two points.
- x. A diameter bisects a circle.
- y. Only one line parallel to a given line passes through a given point.

3-7. Superposition. Two geometric magnitudes that coincide exactly, when one is placed upon the other, are equal. This method of establishing equality is called the method of superposition. In practice you will not actually move the geometric magnitudes but merely compare them mentally. If you conceive that one straight line can be placed upon another straight line so that the ends of both coincide, the lines are equal. If you determine that one angle can be placed over another angle so that their vertices coincide and their sides go in the same directions, the angles are equal. If you determine that one figure can be placed upon another figure so that they coincide at all places, the figures are equal. Figures that coincide exactly when superposed are congruent.

PART E - THEOREMS FOR LINES, ANGLES, AND TRIANGLES

3-8. Basic Concepts of Theorems and Corollaries. A geometric rule that can be proved by using postulates, illustrations, and logical reasoning is called a theorem. A secondary rule whose truth can be easily deducted from a theorem is called a corollary. You can use theorems and corollaries to solve geometric problems without having to use the basic relationships that are established by the postulates.

3-9. Theorems for Lines. The following theorems show the relationship between straight lines in the same plane.

a. Theorem 1. Only one perpendicular line can be drawn from a given line to a given point outside that line. A corollary for this theorem is that a perpendicular line is the shortest line that can be drawn from a given line to a given point.

b. Theorem 2. Two lines in the same plane and perpendicular to the same line are parallel. Therefore, if a straight line is perpendicular to one of two parallel lines, it is also perpendicular to the other. This relationship is shown in Figure 3-12.

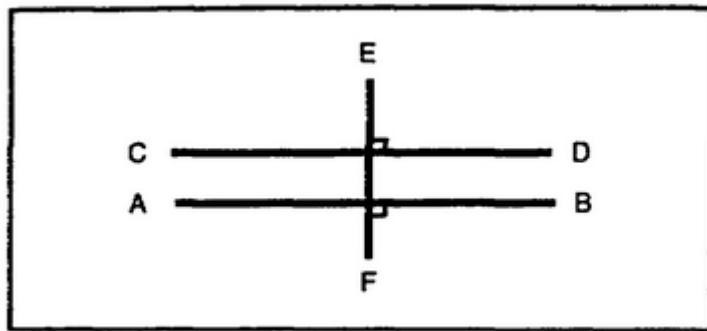


Figure 3-12. Parallel lines cut by a perpendicular line

c. Theorem 3. If two parallel lines are cut by a transversal, the alternate interior angles are equal. Therefore, when two lines are cut by a transversal and one pair of alternate interior angles are equal, the two lines are parallel. In Figure 3-13, lines AB and CD are parallel and line EF is the transversal. Interior angles r and r' are equal, and interior angles s and s' are equal.

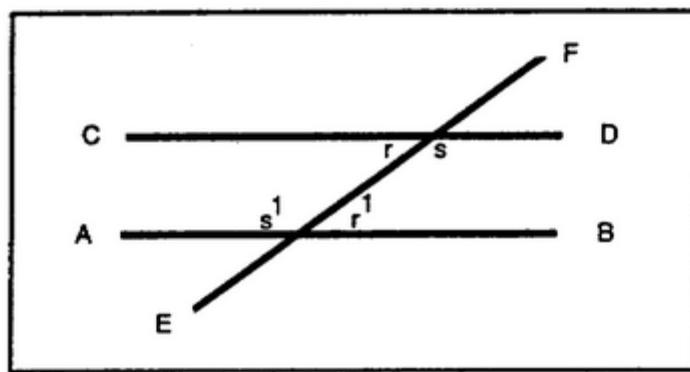


Figure 3-13. Parallel lines cut by a transversal

d. Theorem 4. Straight lines that are parallel to the same line are parallel to each other. In Figure 3-14, if line CD is parallel to AB and line EF is parallel to AB, then line CD is parallel to EF.

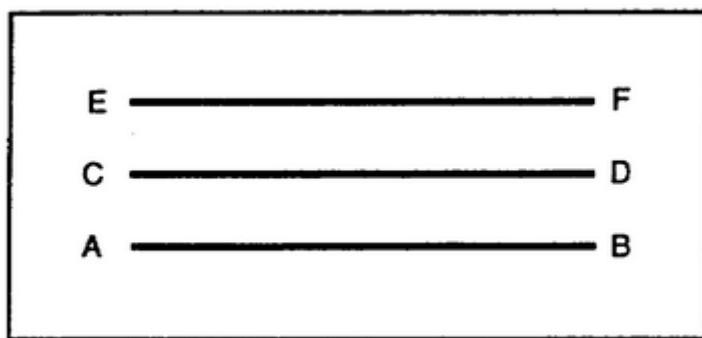


Figure 3-14. Parallel lines

e. Theorem 5. Any point on a perpendicular bisector of a line segment is equidistant from the extremities of the line segment. The distances from any point not on the perpendicular bisector to the

extremities of the line segment are unequal. Figure 3-15 shows that line AC is equal to BC and line AD is equal to BD, but line AE is not equal to BE.

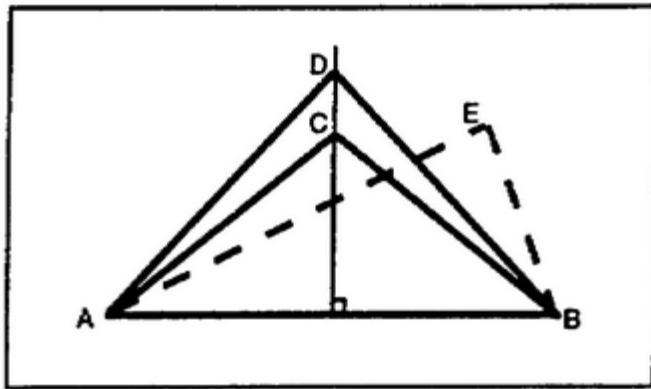


Figure 3-15. Points equidistant from line segment extremities

3-10. Theorems for Angles. The following theorems show the relationship of two angles whose respective sides are either parallel or perpendicular to each other.

a. Theorem 1. If two angles have their sides respectively parallel, the angles are either equal or supplementary, as shown in Figure 3-16. Line C^1C^{11} is parallel with AC, and line A^1B^1 is parallel with AB. Angle $B^1A^1C^1$ is equal to BAC , and angle $B^1A^1C^{11}$ is the supplement of $B^1A^1C^1$; therefore, angle $B^1A^1C^{11}$ is also the supplement of BAC .

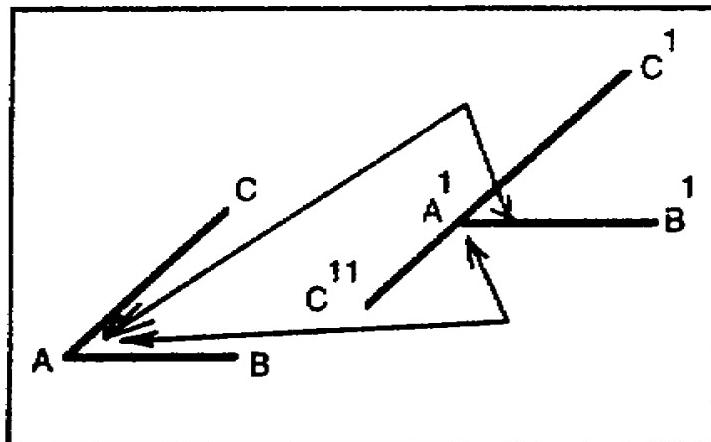


Figure 3-16. Parallel sides

b. Theorem 2. If two angles have their sides respectively perpendicular, the angles are either equal or supplementary, as shown in Figure 3-17. Line C^1C^{11} is perpendicular to AC, and line A^1B^1 is perpendicular to AB. Angle $B^1A^1C^1$ is equal to BAC , and angle $B^1A^1C^{11}$ is the supplement of $B^1A^1C^1$; therefore, angle $B^1A^1C^{11}$ is also the supplement of BAC .

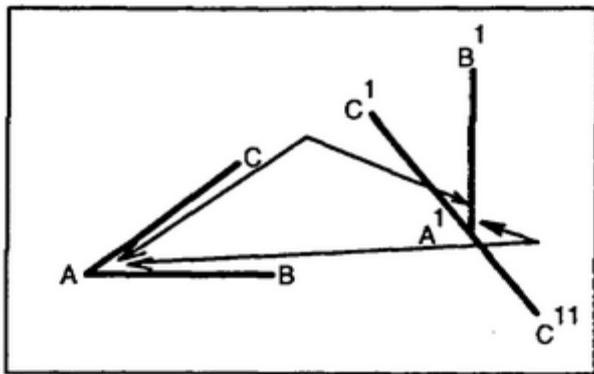


Figure 3-17. Perpendicular sides

c. Theorem 3. Every point on an angle bisector is equidistant from the sides of the angle. The distances from a point not on the angle bisector to the sides of the angle are unequal. Figure 3-18 shows that line AB is equal to BC and line DE is equal to EF, but line GP is not equal to PH. A corollary for this theorem is that all points within an angle that are equidistant from its sides lie in the angle bisector.

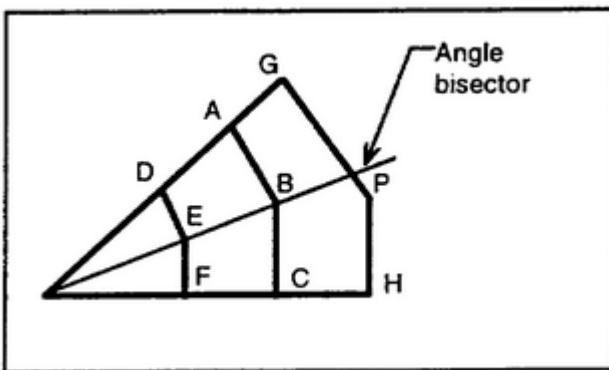


Figure 3-18. Points equidistant from the sides of an angle

3-11. Theorems for Triangles. The following theorems are for the basic relationships of the angles and sides of a triangle.

a. Theorem 1. The sum of the angles of a triangle is equal to two right angles. This theorem can be proved by using the illustration shown in Figure 3-19, page 3-12. Line AB is extended to D, and line BE is parallel to AC. Angle r is equal to w, and angle t is equal to u. Since angle s plus angle u plus angle w equals 180° , angle r plus angle t plus angle s also equals 180° or two right angles. The following corollaries can be developed from this theorem:

- (1) The sum of the two acute angles of a right triangle is equal to a right angle.
- (2) A triangle cannot have more than one right angle or more than one obtuse angle.

(3) The third angles of two triangles are equal if the two angles of one triangle are equal to the two angles of the other triangle.

(4) The right triangles are equal if the side and the acute angle of one triangle are equal to the corresponding side and the acute angle of the other triangle.

(5) Any exterior angle of a triangle is equal to the sum of the two opposite interior angles.

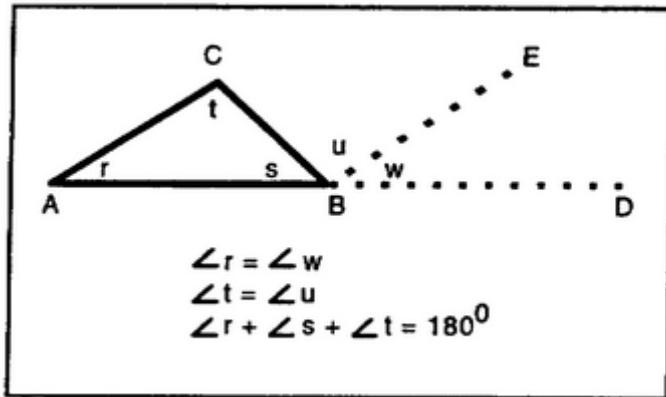


Figure 3-19. Sum of angles equal 180°

b. Theorem 2. Any side of a triangle is less than the sum of the other sides. This theorem can be proved by using the postulate in paragraph 3-6a. As shown in Figure 3-20, side AB is a straight line and the other two sides, AC and CB, form a bent line between points A and B.

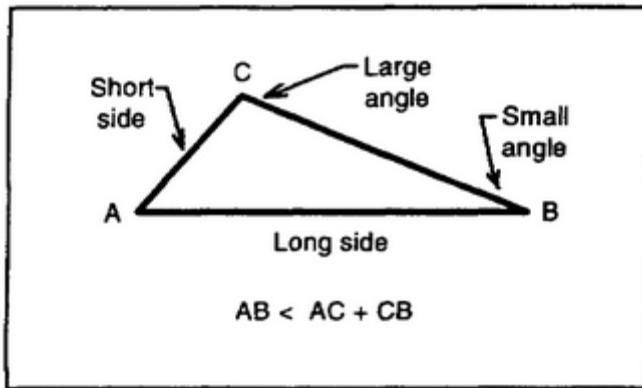


Figure 3-20. Relative sizes of angles and sides

c. Theorem 3. If two sides of a triangle are equal, the angles opposite these sides are equal, as shown in Figure 3-21. Conversely, if two angles of a triangle are equal, the sides opposite these angles are equal. The following corollaries can be derived from this theorem:

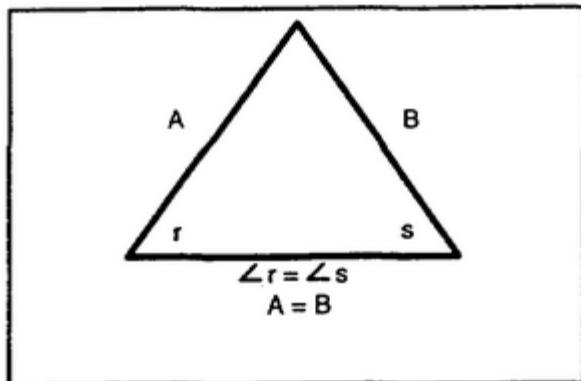


Figure 3-21. Two equal sides

- (1) An equilateral triangle is also an equiangular triangle.
- (2) The bisector of the angle opposite the unequal side of an isosceles triangle is the perpendicular bisector of the base line.
- (3) The perpendicular bisector of the unequal side of an isosceles triangle bisects the opposite angle.
- d. Theorem 3. If one side of a triangle is longer than another side (short side), the angle opposite the long side is greater than the angle opposite the short side. This relationship is shown in Figure 3-20.

3-12. Theorems for Triangle Bisectors, Altitudes, and Medians. The following theorems are for the relationships of the triangle bisectors, altitudes, and medians.

- a. Theorem 1. If a line is parallel to one side of a triangle and bisects the other two sides, it is half as long as the side to which it is parallel. In Figure 3-22, line DC is parallel to AB, line EC is equal to BC, line ED is equal to AD, and line DC is one-half the length of AB.

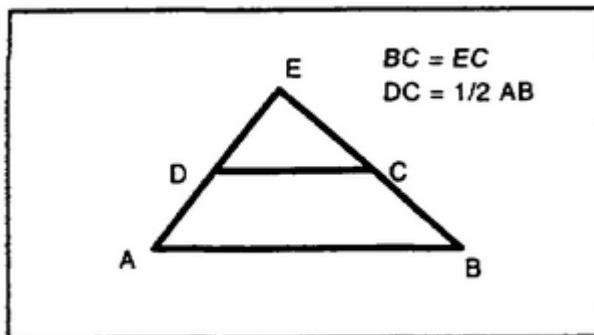


Figure 3-22. Line parallel to one side and bisecting the other two sides

- b. Theorem 2. Perpendicular bisectors of the sides of a triangle meet in a point that is equidistant from the three vertices of the triangle. In Figure 3-23, page 3-14, point P is where the perpendicular bisectors meet.

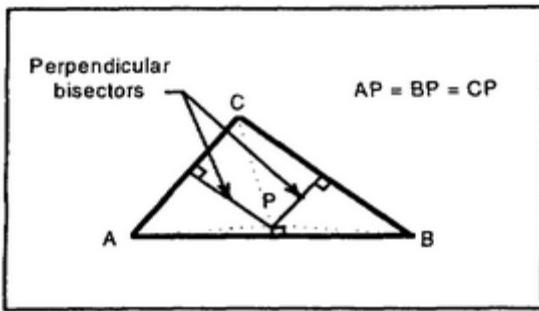


Figure 3-23. Perpendicular bisectors

c. Theorem 3. The altitudes of a triangle meet in a point. In Figure 3-24, point P is where the altitudes meet.

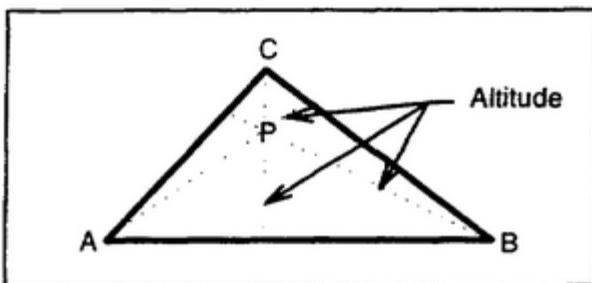


Figure 3-24. Altitudes

d. Theorem 4. The angle bisectors of a triangle meet in a point that is equidistant from the three sides. Figure 3-25 shows that the angle bisectors of a triangle meet at point P. Perpendicular lines from the sides (XP, YP, and ZP) are of equal length.

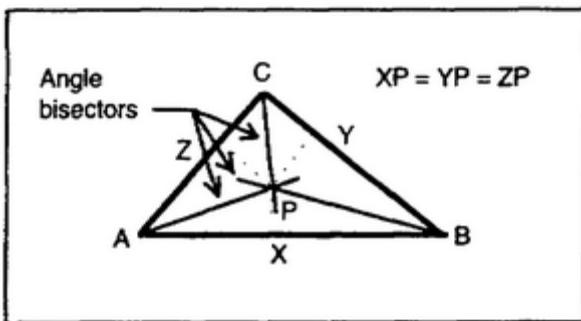


Figure 3-25. Angle bisectors

e. Theorem 5. The medians of a triangle meet at a point that is two-thirds of the distance from each vertex to the midpoint of the opposite side, see Figure 3-26. A median is a line from the vertex to the midpoint of the opposite side of a triangle. Point P is called the centroid of the triangle because this point is the center of gravity.

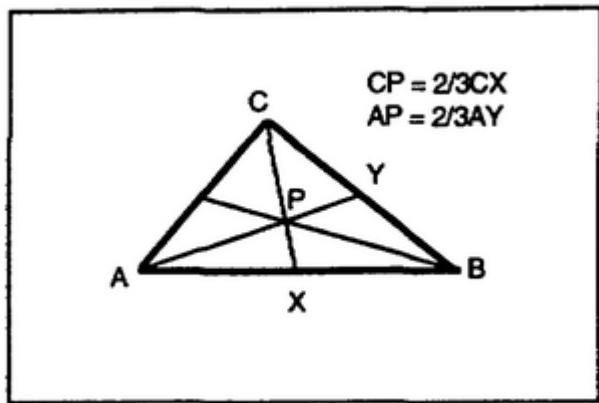


Figure 3-26. Medians

3-13. Theorems for Congruent Triangles. The following theorems can be used to determine whether two triangles coincide exactly when superposed.

- a. Theorem 1. Two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle, as shown in Figure 3-27.

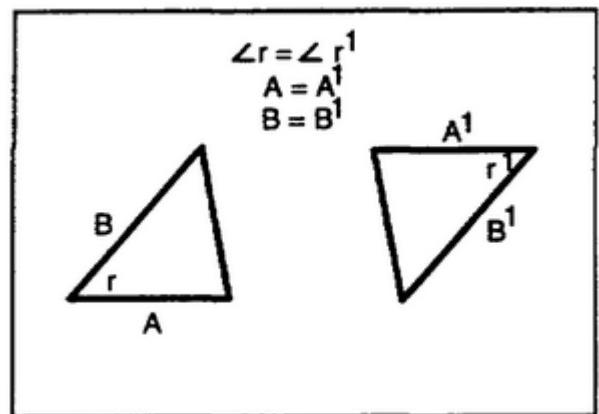


Figure 3-27. Triangles with two sides and the included angle equal

- b. Theorem 2. Two triangles are congruent if a side and the adjacent angles of one triangle are equal to a side and the adjacent angles of the other triangle, as shown in Figure 3-28, page 3-16.

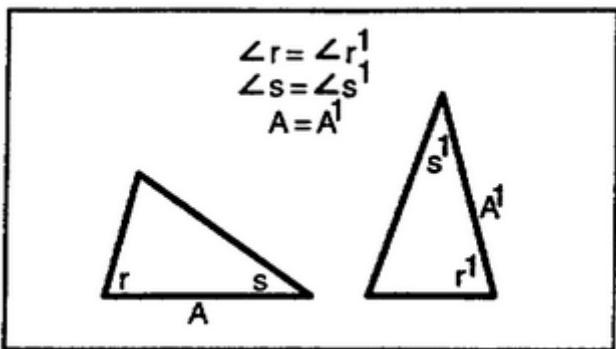


Figure 3-28. Triangles with a side and adjacent angles equal

c. Theorem 3. Two triangles are congruent if the three sides of one triangle are equal, respectively, to the three sides of the other triangle, as shown in Figure 3-29.

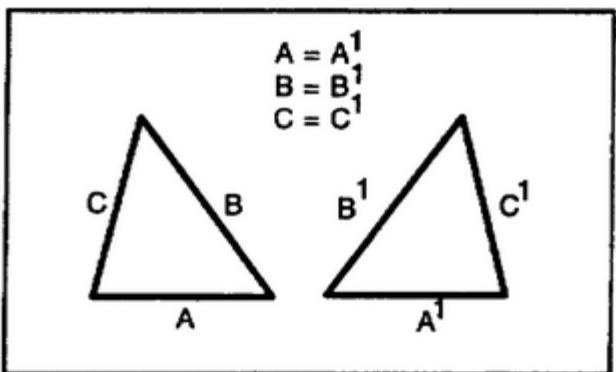


Figure 3-29. Triangles with three sides equal

d. Theorem 4. Two right triangles are congruent if the hypotenuse and a leg of one triangle are equal to the hypotenuse and a leg of the other triangle, as shown in Figure 3-30.

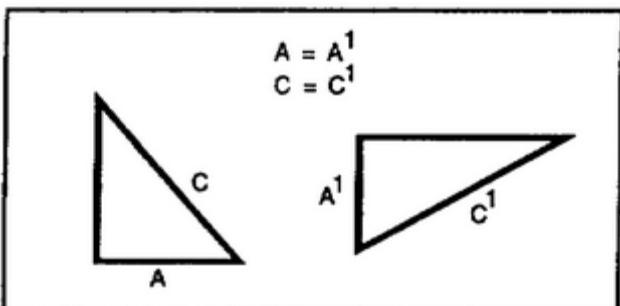


Figure 3-30. Right triangles with two sides equal

3-14. Theorem for the Sides of a Right Triangle. The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. This relationship is known as the Pythagorean theorem.

a. The Pythagorean theorem can be proved by using the illustration shown in Figure 3-31. Examples are as follows:

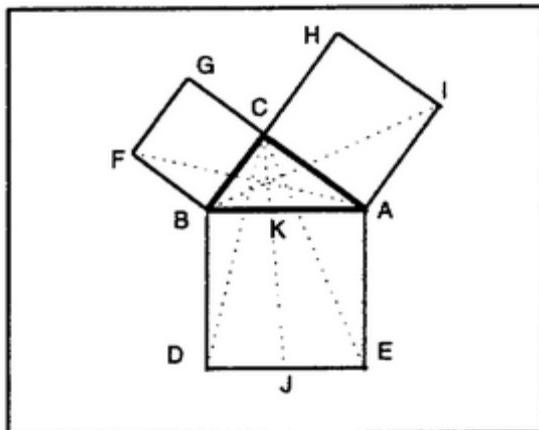


Figure 3-31. Any right triangle

- (1) Line AE is equal to AB and line AI is equal to AC of triangles CAE and IAB because these lines are the sides of the squares.
- (2) Angles EAC and BAI are equal.
- (3) Triangles EAC and BAI are equal.
- (4) AC is the altitude and AI is the base of both square AIHC and triangle AIB.
- (5) The area of triangle AIB is one-half the area of square AIHC.
- (6) AD is the altitude and AE is the base of both rectangle AKJE and triangle ACE.
- (7) The area of triangle ACE is one-half the area of rectangle AKJE. Therefore, the areas of rectangle AKJE and square AIHC are equal.
- (8) Line BD is equal to AB and line BC is equal to BF of triangles CBD and FBA because these lines are the sides of the squares.
- (9) Angles CBD and FBA are equal.
- (10) Triangles CBD and FBA are equal.
- (11) BC is the altitude and FB is the base of both square FBCG and triangle FBA.
- (12) The area of triangle FBA is one-half the area of square FBCG.

- (13) BK is the altitude and BD is the base of both rectangle BDJK and triangle CBD.
- (14) The area of triangle CBD is one-half the area of rectangle BDJK. Therefore, the areas of rectangle BDJK and square FBCG are equal.
- (15) The area of square ABDE is equal to the sum of the areas of the two rectangles (AKJE and BDJK). Therefore, the area of square ABDE is also equal to the sum of the two squares (AIHC and FBCG).

b. A special case of the right triangle, where each side can be divided into an integral number of basic units, is shown in Figure 3-32. The length of side A is three units, the length of side B is four units, and the length of the hypotenuse (side C) is five units. This triangle, called a 3-4-5 right triangle, is often used to mark out long sides with an included right angle. Triangles with sides that are multiples of 3, 4, and 5 (such as 15, 20, and 25) are also considered to be 3-4-5 right triangles. There are other right triangles whose sides can be divided into an integral number of basic units, one of which has sides that are 5, 12, and 13 units long. However, the 3-4-5 triangle is the easiest to remember and the most commonly used.

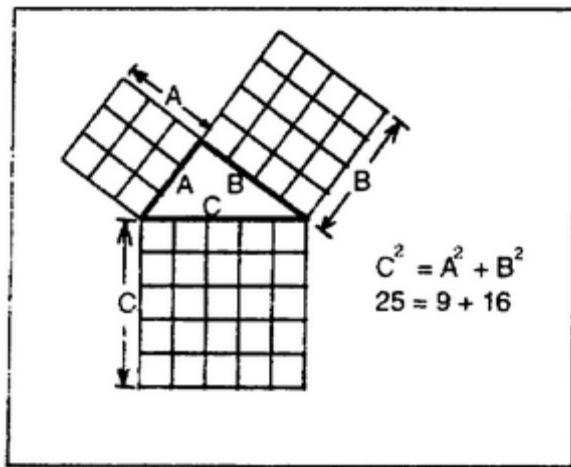


Figure 3-32. Right triangle

3-15. Theorems for Similar Triangles. The corresponding angles of similar triangles are equal. Equal triangles are similar, but similar triangles are not necessarily equal. The important relationship of similar triangles is that a direct proportionality exists between corresponding sides. The following theorems can be used to determine whether triangles are similar.

a. Theorem 1. A line parallel to one side of a triangle and intersecting the other two sides divides these two sides proportionally. This condition is shown in Figure 3-33, in which the following proportion exists:

$$AD : DC = BE : EC$$

Conversely, if a line divides two sides of a triangle proportionally, it is parallel to the third side.

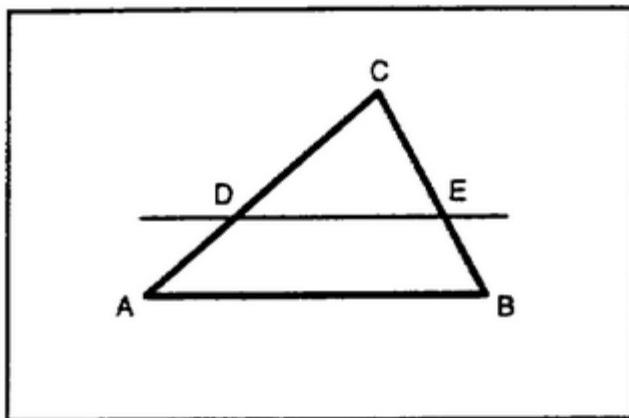


Figure 3-33. Line parallel to one side

b. Theorem 2. If the three angles of one triangle are equal to the three angles of another triangle, the triangles are similar. Similar triangles are shown in Figure 3-34. Since the sum of the interior angles of a triangle is 180° , the following corollaries can be developed from this theorem:

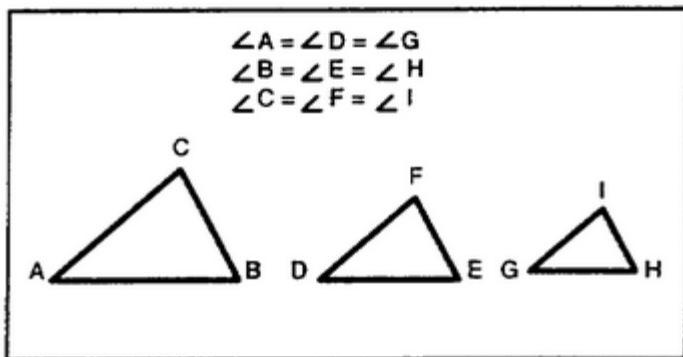


Figure 3-34. Equal angles

(1) Two triangles are similar if two angles of one triangle are equal to the two corresponding angles of the other triangle.

(2) Two right angles are similar if an acute angle of one triangle is equal to an acute angle of the other triangle.

c. Theorem 3. Two triangles are similar if their sides are respectively parallel or if their sides are respectively perpendicular. When this condition exists, the corresponding angles of the two triangles are similar.

d. Theorem 4. In a right triangle, if a perpendicular line is drawn from the vertex of the right angle to the hypotenuse, the resulting two triangles are similar to the original triangle, and they are similar to each other. This relationship is shown in Figure 3-35, page 3-20.

e. Theorem 5. In a right triangle, if a perpendicular line is drawn from the vertex of the right angle to the hypotenuse, the perpendicular line is the mean proportion between the segments of the hypotenuse. The following proportion can then be written for the right triangle shown in Figure 3-35:

$$AD : CD = CD : DB$$

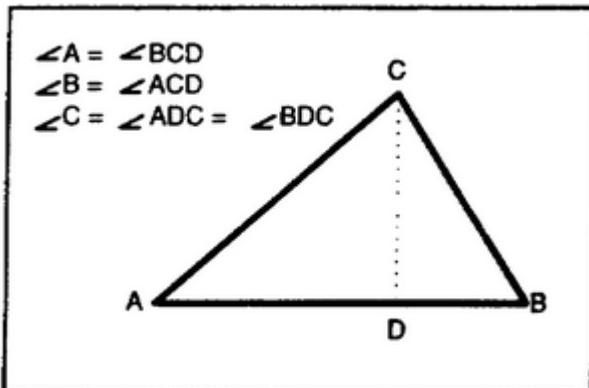


Figure 3-35. Right angles

3-16. Theorem for the Area of a Triangle. The area of a triangle is equal to one-half the product of its base and its altitude. This theorem can be proved with the aid of Figure 3-36, which shows a parallelogram (ABCD) whose sides are parallel to two sides of triangle ABC. Triangles ABC and ACD are equal; therefore, the area of triangle ABC is one-half the area of parallelogram ABCD. The area of parallelogram ABCD is the product of triangle ABC's base and altitude.

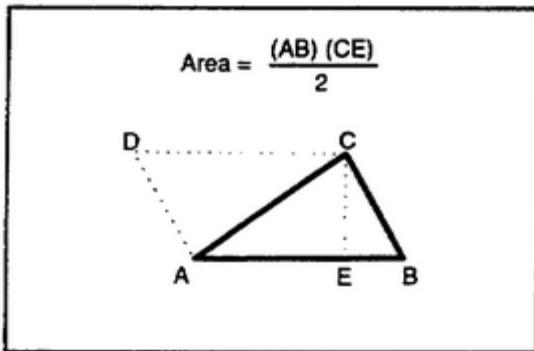


Figure 3-36. Area of a triangle

3-17. Equating like quantities. By using the theorems for similar triangles, you can often determine distances that cannot be readily measured directly. For example, the antenna reflector in Figure 3-37 casts a shadow on the ground that is 35 feet 9 inches from a point directly beneath the antenna pole. At the same time, a yardstick in a vertical position casts a shadow that is 1 foot 2 inches in length. Since light from the sun strikes both the antenna pole and the yardstick at the same angle, angles C and C¹ are equal, and triangles ABC and A¹B¹C¹ are similar. Using the skills of equating like qualities that you learned in Lesson 2 under Ratios and Proportions, set up the problem as follows:

$$\frac{h}{s} = \frac{h^1}{s^1}$$

Then, substituting known values and solving for h:

$$\begin{aligned}\frac{h}{35.75} &= \frac{3 \times 12}{(12 + 2)} \\ h &= \frac{35.75 \times 36}{14} \\ h &= 92 \text{ feet}\end{aligned}$$

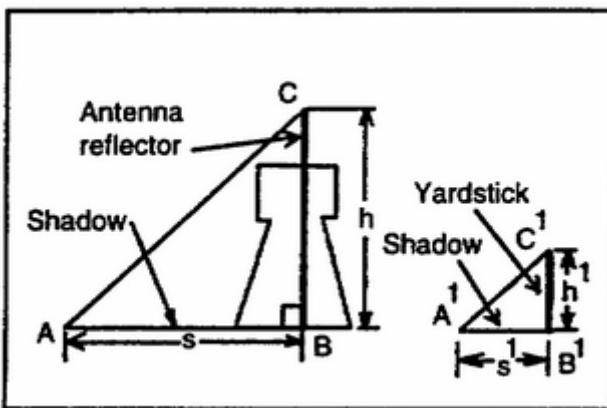


Figure 3-37. Height of antenna reflector

PART F - REVIEW OF GEOMETRY

3-18. Reviewing Geometric Principles. While the actual use of geometry in solving surveying problems is somewhat limited, many of its terms, postulates, theorems, and corollaries have an important bearing on trigonometric computation. The triangle, as you know, plays a very important part in the profession of surveying. For that reason, the following paragraphs will reference you back to pertinent portions of the text in this lesson.

a. In paragraph 3-1, various terms useful in solving geometric problems were defined. Among these were a point, lines (both straight and curved), and flat surfaces. Careful analysis showed that both lines and surfaces were actually generated by a point moving in a straight or a constantly changing direction. In one sense, a line could be considered as the edge view of a flat surface; thus, a straight line could very well be one edge of a flat plane while a curved line could represent one edge of a curved surface.

In plane geometry, a flat plane can be developed by moving a straight line in a direction 90° to its length or by fixing one end of the straight line and swinging its free end a designated distance. By such controlled movement, a segment of a circle that closely resembles a triangle or a complete circle (both of which are flat planes) can be generated from a straight line. The distance that such a straight line is allowed to swing in an arc is measured in special units which are known as degrees, minutes, or seconds. Each of these measurement units is useful when measuring or determining the size of an angle.

b. An angle is ordinarily thought of as being formed by the junction of two straight lines that may or may not continue beyond the junction. If the lines do continue, then the two opposite angles at the vertex are known as vertical angles. This situation was illustrated in Figure 3-7. When the two lines intersect each other at 90° , then four equal angles are formed and are known as right angles. The two right angles sharing a common side and vertex are known as supplementary angles. The opposite or exterior sides of the two right angles form what is known as a straight angle. Each pair of right angles, sharing a common side, can also be called adjacent angles.

c. To solve many of the geometric problems, certain principles have to be taken as they stand and without proof. Based on personal observation, these principles or postulates should be firmly fixed in your mind. There are 25 postulates in all, and they are listed in paragraph 3-6. To aid you in analyzing geometric problems, you can use the superposition method, which allows you to compare by superposition two lines, angles, or figures to determine whether they are equal or congruent.

d. Solving geometric problems is largely based on using theorems and corollaries. The latter term refers to a secondary rule whose truth can be readily derived from a theorem. For convenience, the theorems that are commonly used in plane geometry have been grouped together. Thus, the previous text includes eight theorems which are essential to solving problems that involve lines and angles. These eight theorems are listed under paragraphs 3-9 and 3-10; other theorems that aid in solving triangles are given in paragraphs 3-11 through 3-16.

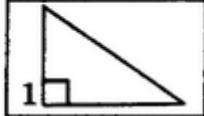
LESSON 3

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. When you have completed the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. If side A of a triangle is 18 units, side B is 24 units, and side C is 30 units, this triangle is referred to as a
 - A. 95 unit triangle
 - B. Base 5 triangle
 - C. Hypotenuse triangle
 - D. 3-4-5 triangle

2. The small  at point 1 is called _____.
 - A. Point of intersection
 - B. Angle symbol
 - C. Right-angle symbol
 - D. Square unit symbol


3. _____ is generated by a point in motion.
 - A. A plane
 - B. A line
 - C. A angle
 - D. A surface

4. Fundamental principles of geometry that depend on personal observation and must be taken without proof are called
 - A. Postulates
 - B. Theorems
 - C. Theories
 - D. Assumptions

5. Lines that bound a triangle are called
 - A. Sides
 - B. Interior and exterior angles
 - C. Bisectors
 - D. Reentrant angles

LESSON 3

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

- | | | |
|------|-----------------------------|--|
| Item | Correct answer and feedback | |
| 1. | D | 3-4-5 triangle
A special case of...(page 3-18, para 3-14b) |
| 2. | C | Right-angle symbol
The small square at...(page 3-3, para 3-2a and Figure 3-4) |
| 3. | B | A line
A line is generated by...(page 3-1, para 3-1) |
| 4. | A | Postulates
Fundamental principles of geometry...(page 3-7, para 3-6) |
| 5. | A | Sides
The lines that bound the...(page 3-5, para 3-5) |

LESSON 4

TRIGONOMETRY

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will learn how to use trigonometry.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will be taught how to use trigonometry in surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, CDC 3E551A, and Appendix C of this ACCP.

INTRODUCTION

Surveyors work almost daily with the triangle in surveying vast expanses of land. In doing so, they use a network of triangles, which is known as triangulation. The sides and angles of a plane triangle are so related that given any three parts, provided at least one of them is a side, the shape and size of a triangle can be determined. This science, which is called trigonometry, is both geometric and algebraic in nature. This branch of mathematics deals with computing unknown parts. It begins by showing the mutual dependence of the sides and angles in a triangle and, for that purpose, employs the ratio of the sides in a right triangle. It is based on the properties of similar triangles and is applied whenever angles enter into the solution of the problem. Because trigonometry deals primarily with angles, it is necessary for the surveyor to have a clear conception of the meaning and the measurement of angles.

PART A - ANGLES

4-1. Defining Angles. An angle is defined as the figure formed by the intersection of two lines at one point. The point is called the vertex of the angle. The two lines forming the angle are called the sides or legs of the angle. The angle, as it applies to trigonometry, is read by designating the capital letter placed at the vertex. The mathematical symbol for the word angle is simply a small \angle . The size of magnitude of an angle is determined by

the difference in direction of the two sides. This size is measured in degrees ($^{\circ}$), minutes ('), and seconds (").

a. The minutes are subdivisions of a degree. There are $60'$ in 1° . The seconds are a subdivision of the minutes. There are $60''$ in $1'$ and $3,600''$ in 1° . Seconds are usually subdivided into tenths and hundredths of a second and are always expressed as decimal fractions of a second.

b. The basis of all angles is the circle, which contains 360° . The four ways 360° may be written are as follows:

- 360°
- $360^{\circ} 00' 00''$
- $359^{\circ} 59' 60''$
- $0^{\circ} 00' 00''$

It should always be remembered that each of the minutes and seconds columns must have at least two figures; therefore, many times a zero must be added in front for a single digit or two zeros must be added to denote no minutes or seconds.

4-2. Computing Angles. The division of an angle is done by dividing the number into degrees, minutes, and seconds. The important point to remember in the first portion of this process is that 1° equals $60'$ and when there are degrees remaining after the initial division, they must be converted to minutes before carrying them over to the minutes column. As shown in the example below, 3 will go into 10° 3 times, leaving a remainder of 1° . Before making the second division, this 1° must be changed to $60'$ and added to the number of minutes in the original problem.

a. The original problem shows $32'$. Add $60'$ (the remainder of 1°) to $32'$, which results in a sum of 92. Divide 92 by 3, which will go 30 times with a remainder of $2'$.

Example: Divide $10^{\circ} 32' 14''$ by 3.

Solution:

$$\begin{array}{r}
 03^{\circ} \quad 30' \quad 44.7'' \\
 \hline
 10^{\circ} \quad +32' \quad +14'' \\
 \hline
 9^{\circ} \quad =92' \quad =134'' \\
 \hline
 90 \quad -\quad 12 \\
 \hline
 12 \quad -\quad 12 \\
 \hline
 14 \quad -\quad 12 \\
 \hline
 2
 \end{array}
 \qquad
 \begin{array}{r}
 0.66 \\
 3 \overline{) 2.00} \\
 \quad 1.8 \\
 \quad .20
 \end{array}
 \qquad
 \text{.66 Rounded = .7}$$

The diagram illustrates the conversion of 10° into minutes. A circle with a '1' is placed under the tens digit '1' of '10°'. An arrow points from this circle to another circle containing a 'P', which stands for 'degrees'. Another arrow points from the '1' circle to a circle containing a '60'', representing the conversion factor of 60 minutes per degree. Below the '10°' is a horizontal line with '60'' written under it, indicating the result of the conversion.

b. Since there are 60" in 1 minute, this same process must be repeated for any remaining minutes after the second division. Therefore, change the 2' remainder to 120" and add it to 14" (the seconds shown in the original problem), which results in a sum of 134". Divide 134" by 3, which will go 44 times with a remainder of 2". By converting this remainder to a decimal fraction (2/3 of a minute equals 0.67 or rounding off to 0.7), you arrive at the answer desired. If there are no degrees or minutes to be carried over, straight division is performed.

c. There will be times when the same angle must be used in field computations a number of times. The simplest way to multiply an angle by a number is to perform each of the following steps:

- Multiply the seconds by the number to determine the total number of seconds.
- Multiply the minutes by the number to determine the total number of minutes.
- Multiply the degrees by the number to determine the total number of degrees.

Example: Multiply $13^\circ 59' 36''$ by 4.

Solution:

$$\begin{array}{r}
 13^\circ & 59' & 36'' \\
 \times 4 \\
 \hline
 52^\circ & 236' & 144'' \\
 \hline
 3 & 2' & \\
 \hline
 55^\circ & 238' & 24'' \\
 \hline
 60 \overline{)238'} & 60 \overline{)144''} & \\
 180 & 120 & \\
 \hline
 55'' & 58' & 24'' \\
 \hline
 \end{array}
 \quad \text{Solution} \rightarrow 55^\circ 58' 24''$$

d. If there are more than 59 seconds in the total seconds, the minutes must be extracted from the seconds. This is done by dividing 60, the number of seconds in 1 minute, into the total number of seconds. The quotient represents the number of minutes contained in the seconds, and the remainder represents the odd number of seconds. This process is repeated to extract the degrees from the total minutes if there are more than 59 after adding the minutes found in the seconds column. If there are any degrees contained in the minutes after they have been extracted, they are added to the total amount of degrees previously determined.

4-3. Converting Angles. Converting degrees, minutes, and seconds to degrees and decimal parts of a degree is done in the following manner:

- Divide the seconds by 60 to convert them to decimal parts of a minute.

- Add the quotient to the minutes, and divide this sum by 60 to convert it to decimal parts of a degree.
- Add the quotient to the degrees; the sum being the converted form of degrees, minutes, and seconds.

Example: Convert $23^\circ 13' 45''$ to a decimal fraction.

Solution:

$$\begin{array}{r}
 23^\circ 13' 45'' \\
 \hline
 .75 \\
 60 \overline{)45.00} \\
 42\ 0 \\
 \hline
 3\ 00 \\
 3\ 00 \\
 \hline
 0
 \end{array}
 \quad
 \begin{array}{r}
 0.22917 \\
 60 \overline{)13.75} \\
 12\ 0 \\
 \hline
 1\ 75 \\
 1\ 20 \\
 \hline
 550 \\
 540 \\
 \hline
 100 \\
 60 \\
 \hline
 40
 \end{array}
 \quad
 23.22917 \quad \text{Solution} \rightarrow 23.22917^\circ$$

a. The conversion of degrees and decimal parts of a degree to minutes and seconds is done in the following manner:

- Multiply the decimal by 60 to convert it to minutes and decimal parts of a minute.
- Multiply the decimal part of a minute by 60 to convert it to seconds.

The result obtained is degrees, minutes, and seconds.

Example: Convert 32.682 to degrees, minutes, and seconds.

Solution:

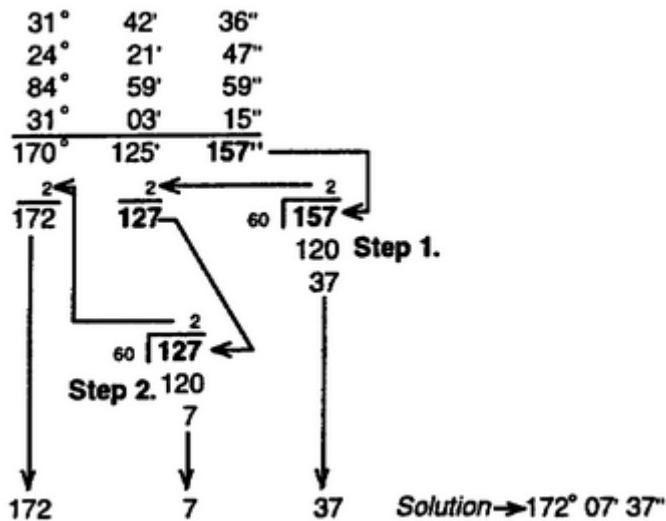
$$\begin{array}{r}
 32.682 \\
 \downarrow \quad \downarrow \quad \downarrow \\
 32 \quad 40 \quad 48
 \end{array}
 \quad
 \begin{array}{r}
 .68 \\
 \times 60 \\
 \hline
 40.80 \\
 \downarrow \quad \downarrow \\
 .80 \\
 \times 60 \\
 \hline
 48.00
 \end{array}
 \quad
 \text{Solution} \rightarrow 32^\circ 40' 48"$$

b. As you will discover, there will be many occasions when two or more angles must be added together. The addition of two or more angles is done in the following manner.

- Add the seconds together.
- Add the minutes together.
- Add the degrees together.

Example: Add the following.

Solution:



c. After the answer has been reached, the minutes must be extracted from the seconds and the degrees must be extracted from the minutes (if there are any) by dividing by 60.

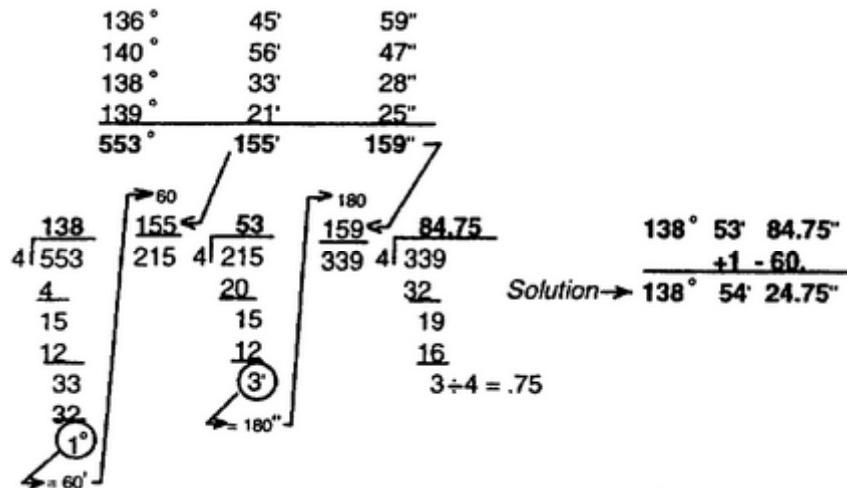
d. The mean angle of two or more angles is determined by combining the processes of addition and division in the following order:

- Add all the degrees, minutes, and seconds.
- Divide the sum by the number of angles.

The quotient is the mean angle.

Example: Find the mean angle.

Solution:



e. Accurately subtracting two or more angles is an important step in computing the length of the forward and back azimuths of a geodetic line based on the known geodetic positions of the ends of the line. This process is known as inverse-position computation. Here are some precautions to observe.

(1) If the minuend is larger than the subtrahend, subtraction is performed. But keep in mind when borrowing degrees or minutes that there are 60' in 1° and 60" in 1', and they must be carried over in this manner.

Example: Subtract 254° 15' 10" from 435° 20' 59".

Solution:

$$\begin{array}{r}
 435^\circ 20' 59" \\
 -254^\circ 15' 10" \\
 \hline
 181^\circ 05' 49"
 \end{array}$$

(2) If the minuend is smaller than the subtrahend, 360° must be added to the minuend. This may be done without actually changing the value of the angle, as may be illustrated by an angle formed by two radii of a circle.

Example: Subtract 94° 59' 59" from 32° 41' 30".

Solution:

The diagram shows the subtraction of angles. The minuend is circled as $32^\circ 41' 30''$ and the subtrahend is circled as $94^\circ 59' 59''$. A horizontal arrow points from the minuend to the equation $360^\circ = 359^\circ 59' 60'' + 360^\circ$. Another horizontal arrow points from the subtrahend to the equation $-94^\circ 59' 59''$. The result is $297^\circ 41' 31''$.

$$\begin{array}{r} 360^\circ = 359^\circ 59' 60'' \\ + 32^\circ 41' 30'' \\ \hline 391^\circ 100' 90'' \\ - 94^\circ 59' 59'' \\ \hline 297^\circ 41' 31'' \end{array}$$

f. If one radius is held stationary and the other is moved 360° , it will return to its original direction without increasing or decreasing the magnitude of the angle. This principle, after adding 360° to the minuend, will allow you to perform straight subtraction.

PART B - TRIGONOMETRIC FUNDAMENTALS

4-4. Functions of Acute Angles. Understanding trigonometric fundamentals is essential to studying surveying, map making, map reading, astronomy, navigation, and many other engineering subjects where objects are represented by drawn figures or are given exact locations. Trigonometry primarily deals with measuring angles and distances. As a surveyor, you will be apply the principles for solving a right triangle to obtain the indirect measurement of angles and distances.

a. In Figure 4-1, note that the triangle's angles are identified by capital letters, and the sides opposite each angle are identified by the same letter only in small print. A general method of identifying the sides of a right triangle is by giving them names in reference to the angles. Referring to the acute angle A in Figure 4-1, side a is known as the side opposite and side b is known as the side adjacent. With reference to the acute angle B, side b is the side opposite and side a the side adjacent. The side opposite the right angle (c in this case) is always known as the hypotenuse.

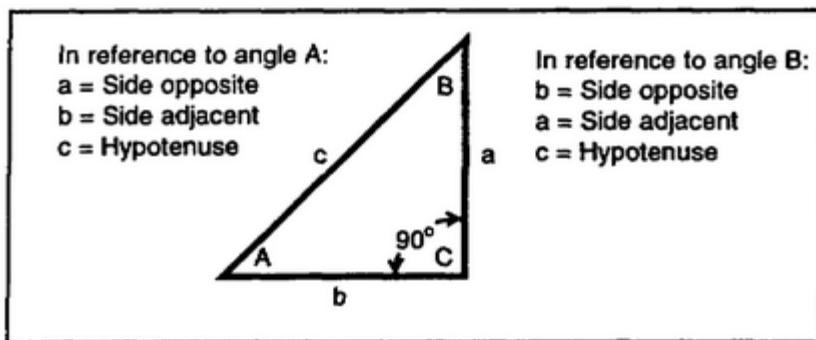


Figure 4-1. Triangle relationships

b. Referring to Figure 4-1, you will see that the following six ratios may be written for the sides of a right triangle: $\frac{a}{c}$, $\frac{b}{c}$, $\frac{a}{b}$, $\frac{c}{a}$, $\frac{c}{b}$, and $\frac{b}{a}$. In Figure 4-2 the sides are extended to produce the triangle $A B^1 C^1$ with sides $a^1 b^1 c^1$. By using the rule of similar triangles, you may set up the following proportion for this triangle.

$$\frac{AB}{AC} = \frac{AB^1}{AC^1}$$

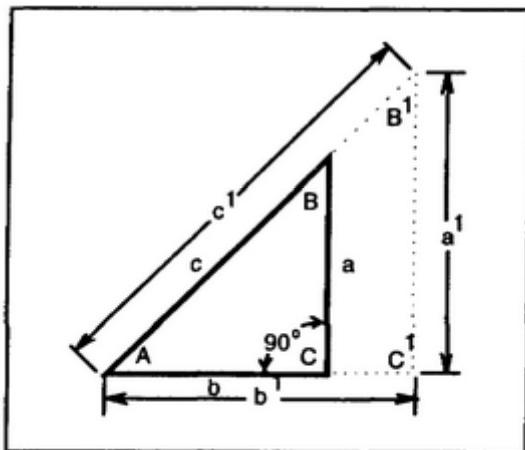


Figure 4-2. Similar triangles

c. Looking at the proportion, note that the difference in the triangle's size does not affect the ratio of the hypotenuse to the base. The ratio is the same for all right triangles with the same acute angle. In any right triangle, the ratio of any two sides depends only on the size of the related acute angle. Therefore, this rule can be applied to the six ratios written above; however, their values depend on the size of the angle.

d. To differentiate between and clearly identify each ratio, note the special names given to them in Table 4-1 (see Figure 4-2 to find the pertinent side and angle). In Table 4-1, the ratio $\frac{a}{c}$ is the sine of A and also the cosine of B. Similarly, $\frac{b}{c}$ is the cosine of A and the sine of B, and the same will hold true for all the other ratios. Therefore, you cannot learn these ratios as $\frac{a}{c}$, $\frac{b}{c}$, $\frac{a}{b}$, $\frac{c}{a}$, $\frac{c}{b}$, or $\frac{b}{a}$, but they must be memorized as $\frac{\text{side adjacent}}{\text{hypotenuse}}$ and so on.

Table 4-1. Special names for ratios

Considering Angle A	Considering Angle B
$\frac{a}{c} = \frac{\text{side opposite}}{\text{hypotenuse}} = \sin A$	$\frac{a}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \cos B$
$\frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \cos A$	$\frac{b}{c} = \frac{\text{side opposite}}{\text{hypotenuse}} = \sin B$
$\frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}} = \tan A$	$\frac{a}{b} = \frac{\text{side adjacent}}{\text{side opposite}} = \cot B$
$\frac{c}{a} = \frac{\text{hypotenuse}}{\text{side opposite}} = \csc A$	$\frac{c}{a} = \frac{\text{hypotenuse}}{\text{side adjacent}} = \sec B$
$\frac{c}{b} = \frac{\text{hypotenuse}}{\text{side adjacent}} = \sec A$	$\frac{c}{b} = \frac{\text{hypotenuse}}{\text{side opposite}} = \csc B$
$\frac{b}{a} = \frac{\text{side adjacent}}{\text{side opposite}} = \cot A$	$\frac{b}{a} = \frac{\text{side opposite}}{\text{side adjacent}} = \tan B$
NOTE:	
Sin = sine	Sec = secant
Cos = cosine	Csc = cosecant
Tan = tangent	Cot = cotangent

4-5. Functions of Trigonometry. If A is any given angle, a set of values can be determined for the six ratios. Because they will vary with the changes in A, the sine A, cosine A, tangent A, cotangent A, secant A, and cosecant A are functions of A and are specifically referred to as trigonometric functions. See Appendix C for the Natural Trigonometric-Functions Tables.

a. In general, the decimal form of a function is an endless decimal. By the use of advanced mathematics, an angle's function can be computed to as many decimal places as desired. In any table of function values, the error in any entry is at most one-half of a unit in the last place. A natural trigonometric-functions table may show 4 to 10 place values and is easy to use. Because of the complementary relationship of the acute angles in a right triangle, that is, sine A = cosine (90-A) and so forth, each entry in a natural trigonometric-functions table serves a dual purpose and, consequently, the table is only one-half as large as it otherwise would be.

b. Figure 4-3, page 4-10, is an example of a natural trigonometric-functions table. From this table, find the sine 32° 20' using the procedures below:

- Locate the page with 32° at the top.
- Find the sin and cos columns under 32°.
- Locate the word minutes at the left of the page. Going down this column, go to 20'.
- Go right across this line to the sin column. There you will find 0.53484, which is the sine of 32° 20'.

32 Degrees					
Minutes	Sin	Cos	Tan	Cot	
0	0.52992	0.84805	0.62487	1.60033	60
1	0.53017	0.84789	0.62527	1.59930	59
2	0.53041	0.84774	0.62568	1.59826	58
3	0.53066	0.84759	0.62608	1.59723	57
4	0.53091	0.84743	0.62649	1.59620	56
5	0.53115	0.84728	0.62689	1.59517	55
6	0.53140	0.84712	0.62730	1.59414	54
7	0.53164	0.84697	0.62770	1.59311	53
8	0.53189	0.84681	0.62811	1.59208	52
9	0.53214	0.84666	0.62852	1.59105	51
10	0.53238	0.84650	0.62892	1.59002	50
11	0.53263	0.84635	0.62933	1.58900	49
12	0.53288	0.84619	0.62973	1.58797	48
13	0.53312	0.84604	0.63014	1.58695	47
14	0.53337	0.84588	0.63055	1.58593	46
15	0.53361	0.84573	0.63095	1.58490	45
16	0.53386	0.84557	0.63136	1.58388	44
17	0.53411	0.84542	0.63177	1.58286	43
18	0.53435	0.84526	0.63217	1.58184	42
19	0.53460	0.84511	0.63258	1.58083	41
20	0.53484	0.84495	0.63299	1.57981	40
21	0.53509	0.84480	0.63340	1.57879	39
22	0.53534	0.84464	0.63380	1.57778	38
23	0.53558	0.84448	0.63421	1.57676	37
24	0.53583	0.84433	0.63462	1.57575	36
25	0.53607	0.84417	0.63503	1.57474	35
26	0.53632	0.84402	0.63544	1.57372	34
27	0.53656	0.84386	0.63584	1.57271	33
28	0.53681	0.84370	0.63625	1.57170	32
29	0.53705	0.84355	0.63666	1.57069	31
30	0.53730	0.84339	0.63707	1.56969	30
31	0.53754	0.84324	0.63748	1.56868	29
32	0.53779	0.84308	0.63789	1.56767	28
33	0.53804	0.84292	0.63830	1.56667	27
34	0.53828	0.84277	0.63871	1.56566	26
35	0.53853	0.84261	0.63912	1.56466	25
36	0.53877	0.84245	0.63953	1.56366	24
37	0.53902	0.84230	0.63994	1.56265	23
38	0.53926	0.84214	0.64035	1.56165	22
39	0.53951	0.84198	0.64076	1.56065	21
40	0.53975	0.84182	0.64117	1.55966	20
41	0.54000	0.84167	0.64158	1.55866	19
42	0.54024	0.84151	0.64199	1.55766	18
43	0.54049	0.84135	0.64240	1.55666	17
44	0.54073	0.84120	0.64281	1.55567	16
45	0.54097	0.84104	0.64322	1.55467	15
46	0.54122	0.84088	0.64363	1.55368	14
47	0.54146	0.84072	0.64404	1.55269	13
48	0.54171	0.84057	0.64446	1.55170	12
49	0.54195	0.84041	0.64487	1.55071	11
50	0.54220	0.84025	0.64528	1.54972	10
51	0.54244	0.84009	0.64569	1.54873	9
52	0.54269	0.83994	0.64610	1.54774	8
53	0.54293	0.83978	0.64652	1.54675	7
54	0.54317	0.83962	0.64693	1.54576	6
55	0.54342	0.83946	0.64734	1.54478	5
56	0.54366	0.83930	0.64775	1.54379	4
57	0.54391	0.83915	0.64817	1.54281	3
58	0.54415	0.83899	0.64858	1.54183	2
59	0.54440	0.83883	0.64899	1.54085	1
60	0.54464	0.83867	0.64941	1.53986	0

Cos Sin Cot Tan Minutes

57 Degrees

Figure 4-3. Example of a natural trigonometric-functions table

c. To read the sine or cosine of an angle greater than 45° , it is necessary to look at the bottom of the page reading up instead of down. The column for minutes will be on the right instead of the left. To find the cosine of $57^\circ 18'$, see Figure 4-3. The cosine of $57^\circ 18'$ is 0.84151. To find the tangent and cotangent of an angle, use the same procedure.

d. Often you will be given the sides of a right triangle, and you will want to find the acute angles of it. Remember that the sine of an angle is equal to the opposite side divided by the hypotenuse. By making a fraction of the two sides' values, you will get the value of the sine function of the angle. To find the angle, look up this value in the proper table under the sin column and then read the number of degrees and minutes in a manner opposite to that used when finding a function for a given angle. For example, if by dividing

the opposite side by the hypotenuse you find the sine value to be $\frac{1.36528}{2}$, or 0.68264, do
the following:

- Locate this number under the sin column in Figure 4-4. You find your angle to be 43° .
- Note that the angle is on the top of the page; therefore, the minute portion of your angle will be found in the minutes column on the left-hand side of the page. Directly on the line to the left of 0.68264 in the minutes column, you find 03'. The final answer then would be $43^\circ 03'$. This problem would be written in the following manner:

$$\sin A = 0.68264$$

$$A = 43^\circ 03'$$

		43	Degrees				
			Minutes	Sin	Cos	Tan	Cot
0	0.68200	0.73135	0.93252	1.07237	60		
1	0.68221	0.73116	0.93306	1.07174	59		
2	0.68242	0.73096	0.93360	1.07112	58		
3	0.68264	0.73076	0.93415	1.07049	57		
4	0.68285	0.73056	0.93469	1.06987	56		
5	0.68306	0.73036	0.93524	1.06925	55		
6	0.68327	0.73016	0.93578	1.06862	54		

Figure 4-4. Finding the acute angle

e. So far you have learned how to find the functions of angles expressed in degrees and minutes. However, it is sometimes necessary to find the functions of angles expressed in seconds. This is done by a process called interpolation. For every change of degrees, minutes, or seconds in an angle, there is a proportional change in the function of the angle. By the use of this proportion, it is possible to interpolate for the seconds of an angle.

f. Suppose you wanted to find the sine $21^\circ 12' 20''$. Since Figure 4-5 shows only the **degrees and minutes, you must interpolate for the seconds. Notice that $21^\circ 12' 20''$ is $\frac{20}{60}$ or one-third of the way from $21^\circ 12' 00''$ to $21^\circ 13' 00''$** , whose functions can be obtained from Figure 4-5.

By the principle of proportional parts discussed in the previous paragraph, an increase of $20''$ in the angle $21^\circ 12'$ causes one-third as much change in its sine as is caused by an increase of 1 minute in the angle.

Minutes	Degree			
	Sin	Cos	Tan	Cot
0	0.35837	0.93358	0.38386	2.60509
1	0.35864	0.93348	0.38420	2.60283
2	0.35891	0.93337	0.38453	2.60057
3	0.35918	0.93327	0.38487	2.59831
4	0.35945	0.93316	0.38520	2.59606
5	0.35973	0.93306	0.38553	2.59381
6	0.36000	0.93295	0.38587	2.59156
7	0.36027	0.93285	0.38620	2.58932
8	0.36054	0.93274	0.38654	2.58708
9	0.36081	0.93264	0.38687	2.58484
10	0.36108	0.93253	0.38721	2.58261
11	0.36135	0.93243	0.38754	2.58038
12	0.36162	0.93232	0.38787	2.57815
13	0.36190	0.93222	0.38821	2.57593
14	0.36217	0.93211	0.38854	2.57371
15	0.36244	0.93201	0.38888	2.57150
16	0.36271	0.93190	0.38921	2.56928
17	0.36298	0.93180	0.38955	2.56707

Figure 4-5. Interpolation

Example: Find the value of sine $21^\circ 12' 20''$. The stated value of any angle is known as its tabular reading.

Solutions:

First Method: Set up the ratio.

$$\begin{aligned}\sin 21^\circ 13' &= 0.36190 \\ \sin 21^\circ 12' &= \underline{0.36162} \\ &\quad 0.00028 \text{ difference}\end{aligned}$$

$$\text{So: } \frac{x}{28} :: \frac{20''}{60''} \text{ or } x = 9$$

Therefore: $\sin 21^\circ 12' = 0.36162$

By interpolation: $20'' = 9$

Then: $\sin 21^\circ 12' 20'' = 0.36171$

Second Method: Take 1/3 tabular difference.

$$\begin{aligned}\sin 21^\circ 13' &= 0.36190 \\ \sin 21^\circ 12' &= \underline{0.36162} \\ &\quad 0.00028 \text{ difference}\end{aligned}$$

$$\text{So: } 1/3 (0.00028) = 0.00009$$

$$\text{Therefore: } \sin 21^\circ 12' = 0.36162$$

$$1/3 \text{ tabular difference} = 0.00009$$

$$\text{Then: } \sin 21^\circ 12' 20'' = 0.36171$$

g. It must be remembered that the tabular values of the sine and tangent increase as the size of the angle increases, and the tabular values of the cosine and cotangent decrease as the size of the angle increases. To avoid confusion and errors in interpolation, it is advisable to follow the methods used above, adding the proportional parts for seconds in the case of sines and tangents and subtracting the proportional parts in the case of cosines and cotangents.

h. As you will recall, the triangle has six parts: three angles (one of which is 90°) and three sides (see Figure 4-6, page 4-14). If two sides, or one side and an acute angle, are given, you can compute the unknown parts of the triangle. This computation is called the solution of a triangle. From this statement, it is evident that in order to solve a right triangle, two parts besides the right angle must be given, at least one of them being a side. The two given parts may be-

- An acute angle and the hypotenuse.
- An acute angle and the opposite leg.
- An acute angle and the adjacent leg.
- The hypotenuse and a leg.
- The two legs.

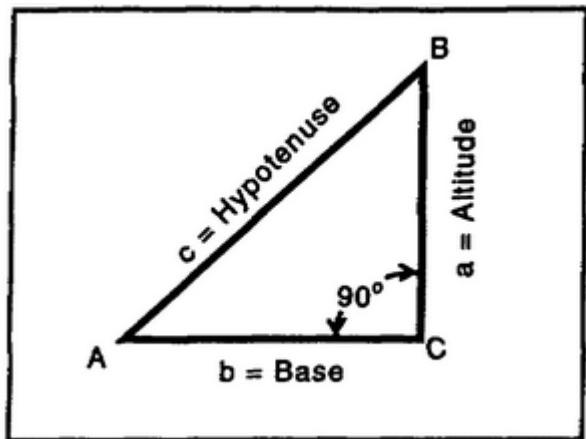


Figure 4-6. A right triangle

i. Before any attempt is made to solve a trigonometric problem, the following steps should be completed.

- Construct a figure as near as possible to scale.
- Letter each given part on the diagram.
- Outline the solution, specifying each triangle and formula to be used.
- Solve each formula for the unknown quantity.

j. To solve the right triangle shown in Figure 4-7, you must find all six parts by following the steps above.

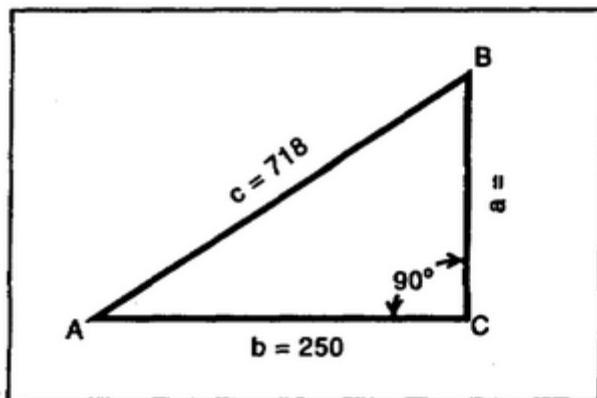


Figure 4-7. Sketch of problem

Example: Solve the right triangle.

Solution:

Known Quantities	Unknown Quantities	Formulas to be used
$b = 250$	$\angle B$	$B = 90^\circ - A$
$c = 718$	$\angle A$	$\cos A = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$
side a		$\tan A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$

Formula for checking: $a = c \cos B$

Solution: Determine A $\cos A = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{250}{718} = 0.34819$

$$\begin{array}{l} \overline{60} \quad \overline{x} \\ \overline{69^\circ 37' 00''} = \end{array}$$

$\cos 69^\circ 37' 00'' = 0.34830$
 $\cos A = 0.34819$
 $\cos 69^\circ 38' 00'' = 0.34803$

$$\begin{array}{l} \overline{27} \\ \overline{1} \\ \overline{0} \end{array}$$

Taken from Figure 4-8

$$\frac{x}{60} = \frac{11}{27} = 11 \text{ divided by } 27 = .4074 \text{ multiplied by } 60 = 24.4 \text{ or } 24$$

then: $A = 69^\circ 37' 24''$

Determine B:
 $B = 90^\circ - A$
 $90 = 89^\circ 59' 60''$
 $69^\circ 37' 24''$
 $20^\circ 22' 36''$

Then: $B = 20^\circ 22' 36''$

Determine a:
 $a = b \tan A =$
 $250 \tan 69^\circ 37' 24''$

$$\begin{array}{l} \overline{60} \quad \overline{x} \\ \overline{69^\circ 37' 00''} = 2.69131 \\ \overline{69^\circ 37' 24''} = ? \\ \overline{69^\circ 38' 00''} = 2.69371 \end{array}$$

$$\frac{24}{60} = \frac{x}{.0024} = 24 \text{ divided by } 60 = .4 \text{ multiplied by } .0024 = .00096$$

$$\begin{array}{l} 2.69131 \\ .00096 \\ \hline 2.69227 \end{array}$$

$$a = b \tan A = 250 \text{ multiplied by } 2.69227 = 673.067$$

Then $a = 673.07$

Taken from Figure 4-9, page 4-16

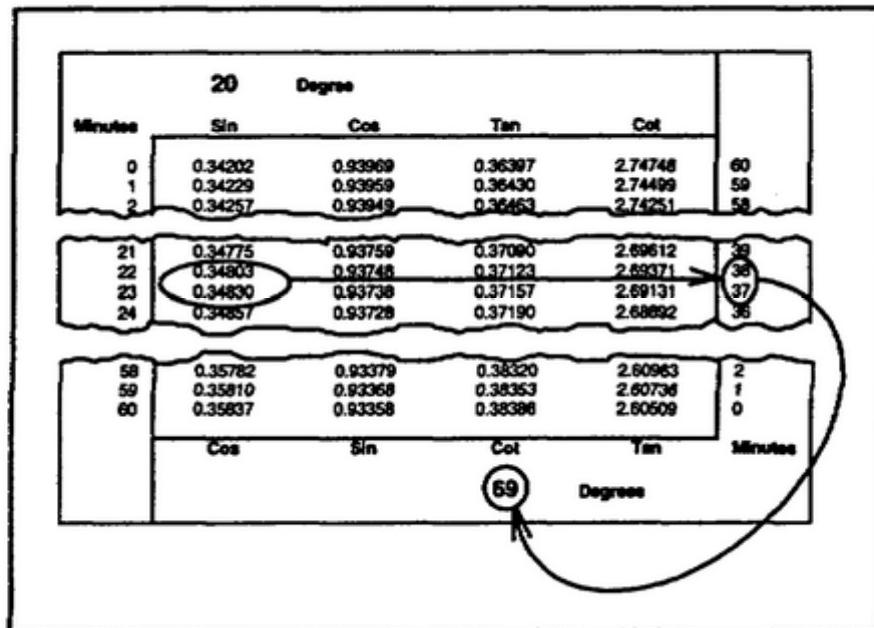


Figure 48. Sines and cosines

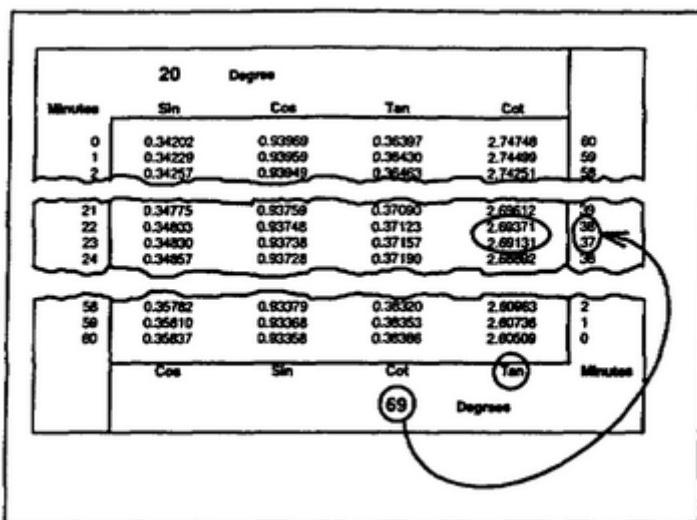


Figure 4-9. Tangents and cotangents

4-6. Functions of Obtuse Angles. Previously, you studied the functions of the acute angles of a right triangle. As a surveyor in the field, you are often required to turn oblique angles, that is angles greater than 90° . It is necessary for you to know the functions of angles greater than 90° in order to solve an oblique triangle. If one of the angles of a triangle exceeds 90° , you no longer have a right triangle. It is possible to express the functions of angles greater than 90° as ratios of the sides of right triangles, and relations can be derived between such functions and the functions of acute angles.

a. To determine the functions of angles greater than 90° , you will construct a pair of straight lines intersecting at right angles, as shown in Figure 4-10. The intersection point of the two lines is called the origin and is labeled A. The horizontal line is labeled XX^1 , and the perpendicular line is labeled YY^1 . Using these lines as axes of a rectangular coordinate system, a point (such as B) can be located by its coordinates x and y. The x coordinate is positive when B is to the right of YY^1 and negative when B is to the left of YY^1 . The y coordinate is positive when B is above XX^1 and negative when B is below XX^1 . Using the origin as center and any convenient radius, you can construct a circle that will enable you to determine the functions of angles from 0° through 360° . This is shown in Figure 4-11.

b. Referring to Figure 4-11, you see that the circle is divided into four equal parts, each containing 90° of arc. These parts are called quadrants and are labeled counterclockwise from the line AX as follows:

- The first quadrant is contained in the segment of the circle XAY and is lettered with the Roman numeral I. It contains the angles between 0° and 90° .
- The second quadrant is contained in the segment of the circle $YAXP$ and is labeled II. It contains the angles between 90° and 180° .

- The third quadrant is contained in the segment of the circle X^1AY^1 and is lettered III. It contains the angles between 180° and 270° .
 - The fourth quadrant is contained in the segment of the circle Y^1AX and is lettered IV. It contains the angles between 270° and 360° .
- c. If you begin at the line AX and rotate a radius through 360° , you can construct any number of right triangles in each of the four quadrants shown in Figure 4-11. The radius will represent the hypotenuse of the right triangle, a perpendicular line from the intersection of the radius and the circle to

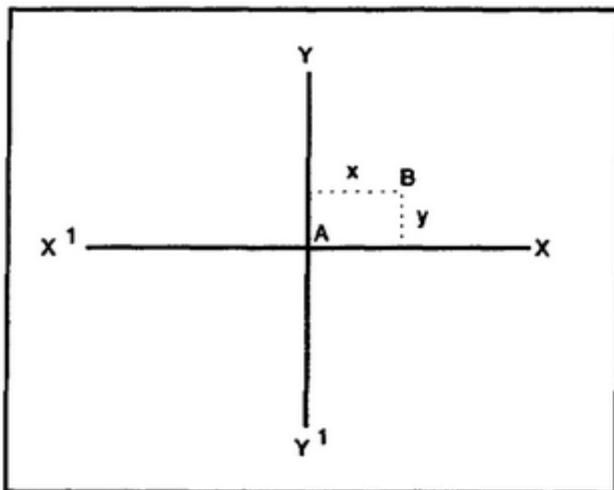


Figure 4-10. The coordinate system

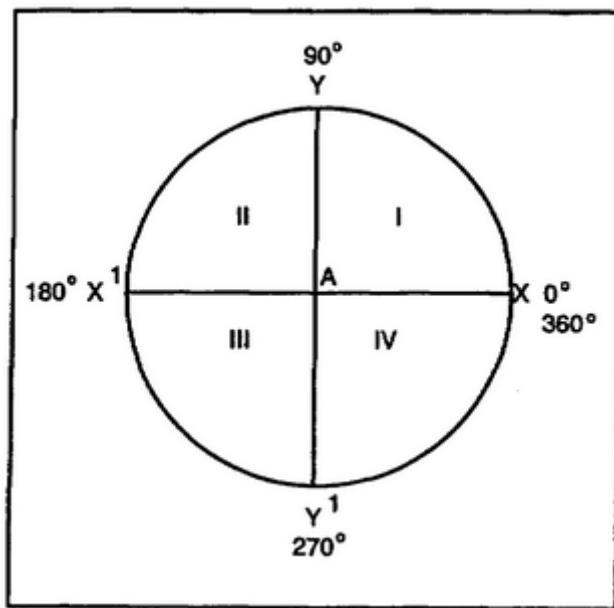


Figure 4-11. Quadrants of a circle

the line X^1X will be the side opposite the angle, and the distance from the center of the circle to where the perpendicular line intersects the line X^1X will be the side adjacent. Note that as the angle increases, the side opposite the side adjacent decreases until at 90° the side opposite is a radius and coincides with the line AY . As the angle increases from 90° to 180° , the side opposite will decrease and the side adjacent becomes a radius and the side opposite is 0° . From 180° to 270° , the side opposite increases and the side adjacent decreases until at 270° the side opposite is again a radius of the circle and the side adjacent is 0° . From 270° to 360° , the side opposite decreases and the side adjacent increases until at 360° the two sides are in the position of an angle of 0° . From the above, you can see that the side opposite can be expressed as a y distance and the side adjacent as an x distance.

d. Since the side opposite is essentially a y distance, you can affix positive signs to the sides opposite of any angle in this circle that lies above the horizontal line X^1X . Similarly, the negative sign can be affixed if the side opposite lies below the line X^1X . Likewise, if an angle has its side adjacent on the right side of the line Y^1Y , the side adjacent will be positive; if it lies on the left side of this line, it will be negative. Applying the above to Figure 4-11, you see that the angles from 0° to 90° have their side opposite above line X^1X and their side adjacent to the right of line Y^1Y . Therefore, all angles in the first quadrant have positive signs affixed to their sides opposite and adjacent. For angles in the second quadrant ($90^\circ - 180^\circ$), the side opposite is still above the line X^1X , but the side adjacent is to the left of line Y^1Y . Angles in quadrant II have positive sides opposite and negative sides adjacent. In the third quadrant ($180^\circ - 270^\circ$), the side opposite is below the line X^1X , and the side adjacent is still to the left of line Y^1Y . These angles have negative sides opposite and adjacent. For angles in quadrant IV ($270^\circ - 360^\circ$), the side opposite is still below the line X^1X , but the side adjacent is again to the right of line Y^1Y . Therefore, the side opposite is negative, and the side adjacent is positive. The hypotenuse of any right triangle is always considered to be positive.

e. It is always possible to express any one of the six trigonometric functions of any angle as a plus or minus trigonometric function of a positive angle less than 90° . Consider the problem of expressing the functions of 220° in terms of functions of an angle less than 90° . Referring to Figure 4-12, observe the following:

$$\sin 220^\circ = \frac{y(\text{of } P)}{r} = \frac{-y}{r} = -\frac{y}{r}$$

$$\sin 40^\circ = \frac{y}{r}$$

$$\therefore \sin 220^\circ = -\sin 40^\circ$$

f. The functions of any angle in the second quadrant ($90^\circ - 180^\circ$) can be found by using the above principles combined with the principles of supplementary angles.

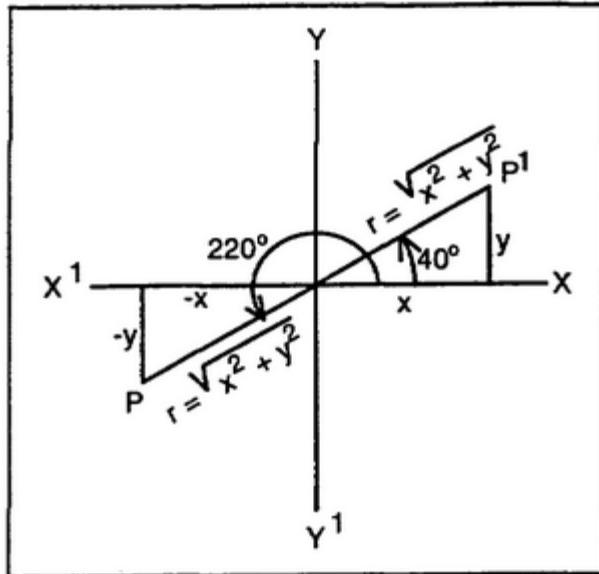


Figure 4-12. Trigonometric function of any angle

g. A supplementary angle is an angle that when added to another angle equals 180° . Since all angles in the second quadrant are larger than 90° , their supplements must be acute angles. Assuming that angle XAB in Figure 4-13, page 20, can have any value from 90° to 180° , you can derive the following formulas for the functions of any angle in the second quadrant.

(1) **Sin of angle XAB:**

$$\sin XAB = \frac{y}{r} = \sin X^1AB$$

$$\therefore \sin XAB = \sin (180^\circ - XAB)$$

(2) **Cos of angle XAB:**

$$\cos XAB = \frac{-x}{r} = -\cos X^1AB$$

$$\therefore \cos XAB = -\cos (180^\circ - XAB)$$

(3) **Tan of angle XAB:**

$$\tan XAB = \frac{y}{-x} = -\tan \frac{X}{AB}$$

$$\therefore \tan XAB = -\tan (180^\circ - XAB)$$

(4) **Cot of angle XAB:**

$$\cot XAB = \frac{-x}{y} = -\cot X^1AB$$

$$\therefore \cot XAB = -\cot (180^\circ - XAB)$$

(5) Secant of angle XAB:

$$\sec XAB = \frac{r}{-x} = -\sec X^1AB$$

$$\therefore \sec XAB = -\sec (180^\circ - XAB)$$

(6) Cosecant of angle XAB:

$$\csc XAB = \frac{r}{y} = \csc X^1AB$$

$$\therefore \csc XAB = \csc (180^\circ - XAB)$$

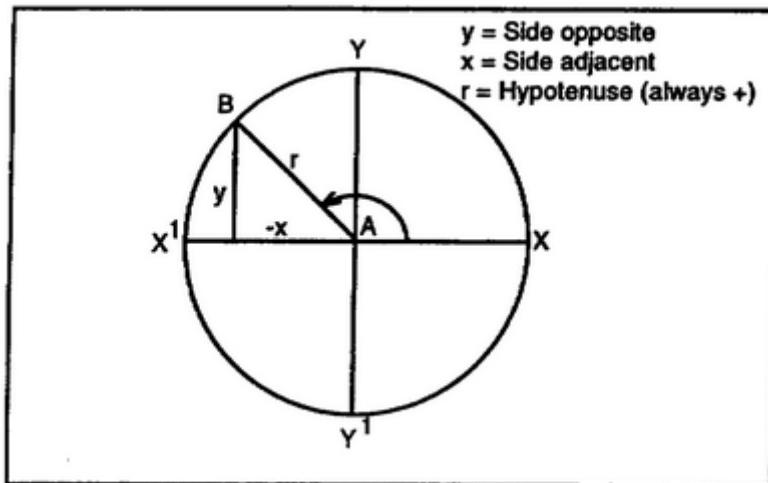


Figure 4-13. Angles between 90° and 180°

h. The functions of any angle between 180° and 270° can be expressed as the function of the acute angle, which is found by subtracting 180° from the given angle. Assuming that angle XAB in Figure 4-14 can have any value from 180° to 270° , you can derive the following formulas for the functions of any angle in the third quadrant.

(1) Sin of angle XAB:

$$\sin XAB = -\sin X^1AB = -\sin (XAB - 180^\circ)$$

(2) Cos of angle XAB:

$$\cos XAB = -\cos X^1AB = -\cos (XAB - 180^\circ)$$

(3) Tan of angle XAB:

$$\tan XAB = \tan X^1AB = \tan (XAB - 180^\circ)$$

(4) Cot of angle XAB:

$$\cot XAB = \cot X^1AB = \cot (XAB - 180^\circ)$$

(5) Sec of angle XAB:

$$\sec XAB = -\sec X^1AB = -\sec (XAB - 180^\circ)$$

- (6) Csc of Angle XAB:

$$\csc XAB = -\csc X^1 AB = -\csc (XAB - 180^\circ)$$

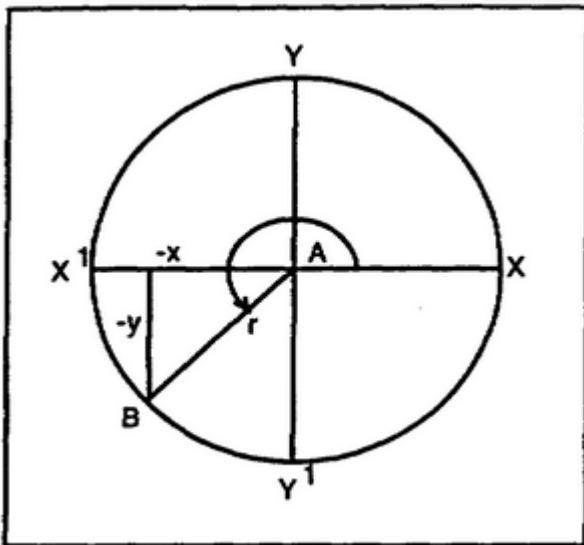


Figure 4-14. Angles between 180° and 270°

i. The functions of any angle between 270° and 360° may be expressed as the function of the acute angle, which is found by subtracting the given angle from 360°. Assuming that angle XAB in Figure 4-15, page 22, can have any value from 270° to 360°, you can derive the following formulas for the functions of any angle in the fourth quadrant.

- (1) Sin of angle XAB:

$$\sin XAB = -\sin (360^\circ - XAB)$$

- (2) Cos of angle XAB:

$$\cos XAB = \cos (360^\circ - XAB)$$

- (3) Tan of angle XAB:

$$\tan XAB = -\tan (360^\circ - XAB)$$

- (4) Cot of angle XAB:

$$\cot XAB = -\cot (360^\circ - XAB)$$

- (5) Sec of angle XAB:

$$\sec XAB = \sec (360^\circ - XAB)$$

- (6) Csc of angle XAB:

$$\csc XAB = -\csc (360^\circ - XAB)$$

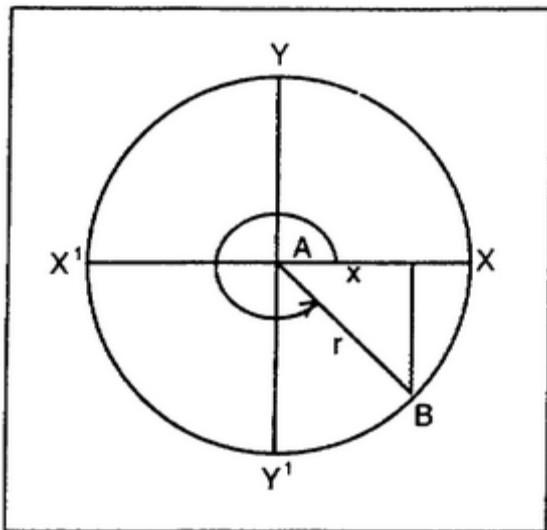


Figure 4-15. Angles between 270° and 360°

- j. Table 4-2 shows the relationship of the sign, the function, and the quadrant of an angle.

Table 4-2. Relationship of the sign, the function, and the quadrant

Functions	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
Sine	+	+	-	-
Cosine	+	-	-	+
Tangent	+	-	+	-
Cotangent	+	-	+	-
Secant	+	-	-	+
Cosecant	+	+	-	-

4-7. The Unit Circle. If a circle has a radius of unity, then the numerical values of the functions for a given angle are represented by the lengths of the lines. This is illustrated for a second-quadrant angle in Figure 4-16.

4-8. Oblique Triangles. Many incidences arise in your daily work that deal with the solution of oblique triangles. To solve these triangles, the fundamental principles of the functions of angles greater than 90° must be understood. The functions of angles greater than 90° can be expressed as the functions of acute angles. These acute angles are found by subtracting the given angle from-

- 180° for quadrant II.
- 270° for quadrant III.
- 360° for quadrant IV.

To each of the functions obtained for these acute angles, you must affix the proper sign as determined from the size of the angle. Although there are numerous ways of numbering the quadrants of a circle, there will be no change in the signs of the functions of each quadrant as shown in Table 4-2.

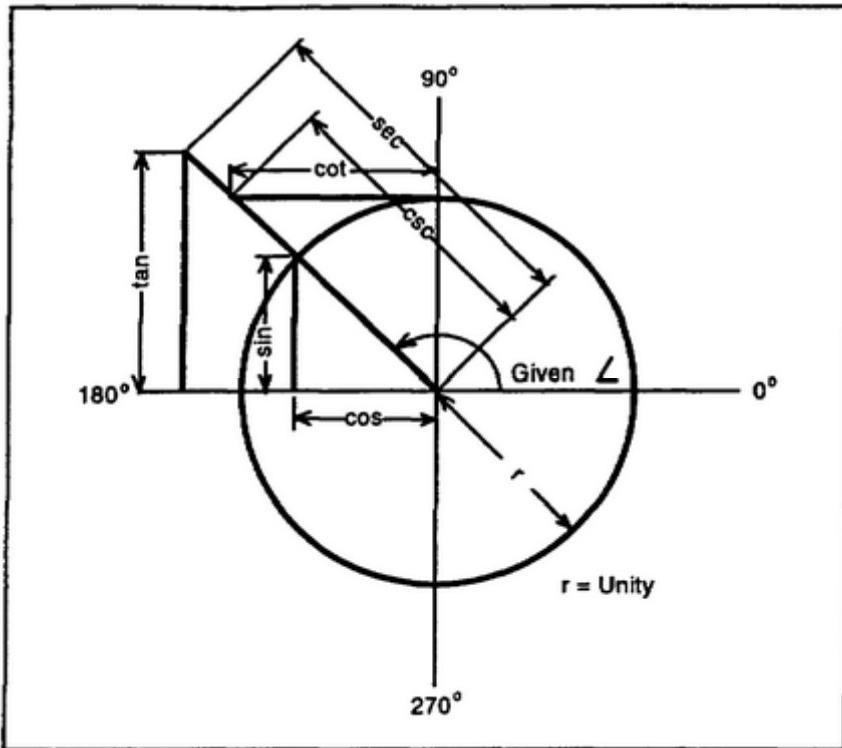


Figure 4-16. The unit circle

a. In previous paragraphs, you dealt with the solution of a special kind of triangle--the right triangle. Since the majority of the angles turned by the surveyor in the field are not right angles, you must know how to solve triangles that are not right triangles. In the following paragraphs, you will review the laws and the methods governing the solution of triangles.

b. It is possible to solve any triangle in which three of its six parts are known, providing one known part is a side. In a right triangle, the right angle is constant; so only a side and one other part must be known to solve the triangle. However, in the oblique triangle, one side and two other parts must be known before the triangle can be solved.

4-9. The Solution of Oblique Triangles. There are four combinations of angles and sides that will supply you with the information necessary for solving an oblique triangle. They are as follows:

- One side and two angles.
- Two sides and an opposite angle.

- Two sides and the included angle.
- Three sides.
 - a. Any triangle can be solved by dividing the triangle into two right triangles and applying the formulas for solving right triangles. To eliminate much of the work involved in solving two right triangles, formulas or laws have been derived that enable you to solve oblique triangles directly.
 - b. The sine law states that in any triangle, the sides are proportional to the sines of their opposite angles. This law also applies in solving triangles where the given information includes one side and two angles or two sides and an opposite angle. When applying the sine law to Figure 4-17, you get various relationships and equations. For example:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

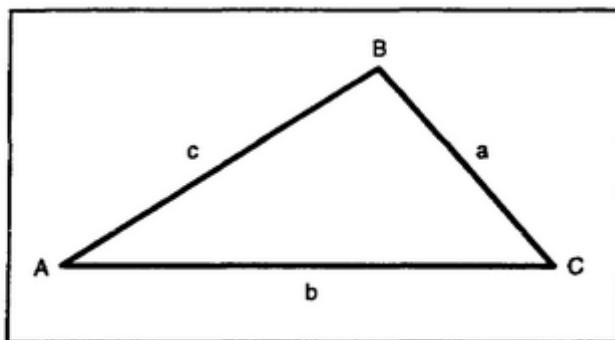


Figure 4-17. The oblique triangle

- c. Since you can solve any triangle in which three parts are known, including a side, it is necessary to set up a relationship between any two sides and their opposite angles. These relationships are as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- d. The cosine law states that in any triangle, the square of any side is equal to the sum of the squares of the other sides minus twice the product of those two sides time the cosine of the angle included between them. This law is adaptable to solving triangles where two sides and the included angle are given and it is desired to find the length of the third side. Three situations exist where the cosine law can be applied. They are as follows:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = b^2 + a^2 - 2ab \cos C$$

If the given included angle is obtuse, remember that its cosine is negative and this sign must be applied when the value of the cosine is substituted in the formula.

e. The tangent law states that the sum of any two sides of a triangle is to their difference as the tangent of half the sum of their opposite angles is to the tangent of half the difference between their opposite angles. This law is best suited for solving triangles in which two sides and the included angle are given. The tangent law in equation form is as follows:

$$\frac{b+a}{b-a} = \frac{\tan \frac{1}{2}(B+A)}{\tan \frac{1}{2}(B-A)}$$

- OR -

$$\tan \frac{1}{2}(B-A) = \frac{b-a}{b+a} [\tan \frac{1}{2}(B+A)]$$

Since it is customary to avoid negative quantities when using the tangent law, you can rearrange the equation to eliminate this. If in the equation above side a had been longer than side b, the equation would have been as follows:

$$\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} [\tan \frac{1}{2}(A+B)]$$

f. The half-tangent formulas are used to solve the angles of a triangle in cases where the given information regarding the triangle is three sides. These formulas are as follows:

$$\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan \frac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\tan \frac{1}{2}C = \sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$$

In the formulas above, s is equal to one-half the summation of the three sides (a, b, and c).

g. In the following paragraphs, there are some typical cases in which the above laws will help you in determining the unknown quantities in oblique triangles.

4-10. Case I - One Side and Two Angles. To solve a triangle where the given information includes a side and two angles, find the value of the third angle by subtracting the sum of the two given angles from 180° . (The sum of the interior angles of a triangle equals 180° .) Then, using the sine law, solve the two unknown sides.

Example: In Figure 4-18, side a of the triangle equals 175, angle A equals $76^\circ 30' 00''$, and angle B equals $48^\circ 45' 00''$. Find angle C and sides b and c.

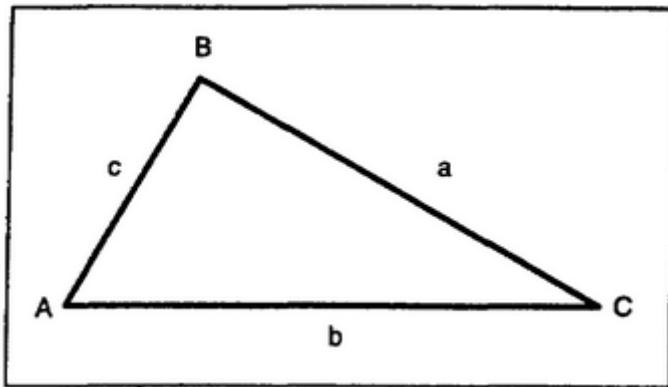


Figure 4-18. Case I

Solution: Find the value of angle C by subtracting the sum of angles A and B from 180° .

$$180^\circ - (76^\circ 30' 00'' + 48^\circ 45' 00'') = 54^\circ 45' 00'' = C$$

Then, using the sine law, solve for the two unknown sides.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$b = \frac{a \sin B}{\sin A} = \frac{(175)(\sin 48^\circ 45' 00'')}{\sin 76^\circ 30' 00''}$$

$$= \frac{(175)(0.75183989)}{0.9723699}$$

$$= 135.3 \text{ (by natural functions)}$$

Also find side c.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{(175)(\sin 54^\circ 45' 00'')}{\sin 76^\circ 30' 00''}$$

$$= \frac{(175)(0.8166416)}{0.9723699}$$

$$= 147.0 \text{ (by natural functions)}$$

Note that the same results are obtained when you use natural functions or logarithms of circular functions.

4-11. Case II - Two Sides and an Opposite Angle. In solving a triangle where two sides and an opposite angle are known, it is necessary to solve for one of the unknown angles using the sine law. Next, you will solve for the third angle by subtracting the sum of the given angle and the solved-for angle from 180°. Then, by using the sine law, you can solve for the remaining unknown side.

Example: In Figure 4-19, side b of the triangle equals 135.2 feet, side a equals 196.6 feet, and angle A equals 32° 36' 40". Find angles B and C and side c.

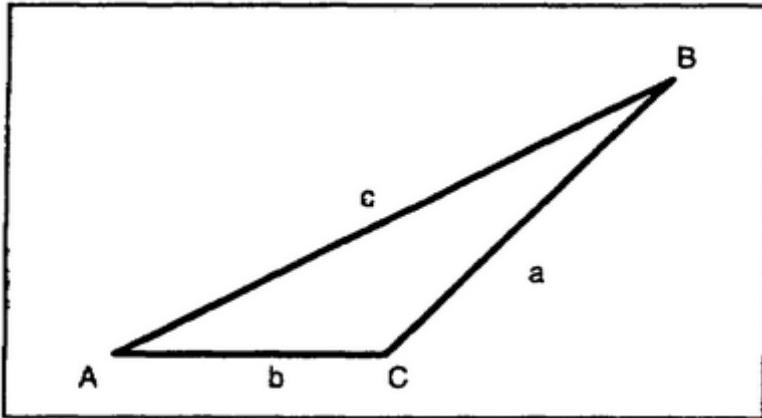


Figure 4-19. Case II

Solution: Using the sine law, solve for angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{196.6}{\sin 32^\circ 36' 40''} = \frac{135.2}{\sin B}$$

$$\sin B = \frac{(135.2)(\sin 32^\circ 36' 40'')}{196.6}$$

$$= \frac{(135.2)(0.539756)}{196.6}$$

$$= 0.3711185$$

$$B = 21^\circ 45' 14'' \text{ (by natural functions)}$$

Then solve for angle C by subtracting the sum of angles A and B from 180°.

$$C = 180^\circ - (32^\circ 36' 40'' + 21^\circ 45' 14'') = 125^\circ 38' 06''$$

Then solve for side c using the sine law.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$c = \frac{a \sin C}{\sin A} = \frac{(196.6)(\sin 125^\circ 38' 06'')}{\sin 32^\circ 36' 40''}$$
$$= \frac{(196.6)(0.8127450)}{0.539756}$$

$$c = 296.03 \text{ feet (by natural functions)}$$

a. The sine of an acute angle is the same as the sine of the supplementary obtuse angle. Therefore, in solving a triangle under case II by the sine law, two values for the angle are possible and either value can be taken unless excluded by other conditions in the triangle.

b. In any triangle, only one of the angles can be obtuse. When the given angle is obtuse, both of the other angles are acute. In Figure 4-20, angle A of the triangle is acute, and side c times the sine of A equals the line BP. This is the side opposite the given angle in the right triangle ABP. It is also the altitude of the triangle in question; therefore, BC or BC' cannot be shorter than the altitude. When side a is less than side c times the sine of A, the triangle is impossible. When angle A is obtuse, side a must be longer than side c or the triangle is impossible. When the angle A is acute and the length of the triangle is shorter than side c but longer than side c times the sine of A (BP), the triangle is ambiguous and two solutions are possible.

4-12. Case III - Two Sides and the Included Angle. When solving for the third side of a triangle and two sides and the included angle are known, it is best to use the cosine law.

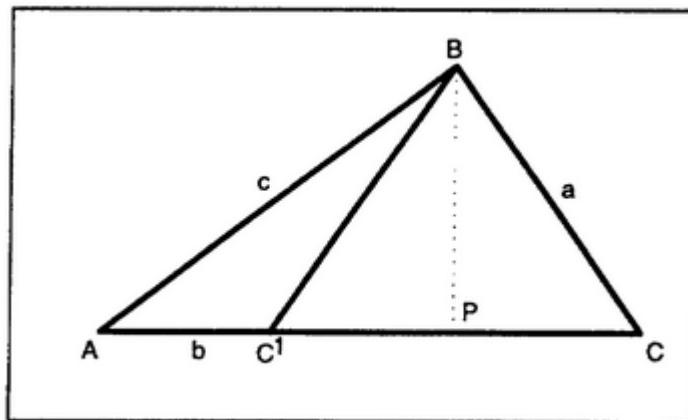


Figure 4-20. The ambiguous triangle

Example: In Figure 4-21, side b of the triangle equals 97.85 feet, side c equals 106.66 feet, and angle A equals $73^\circ 19' 27''$. Find side a.

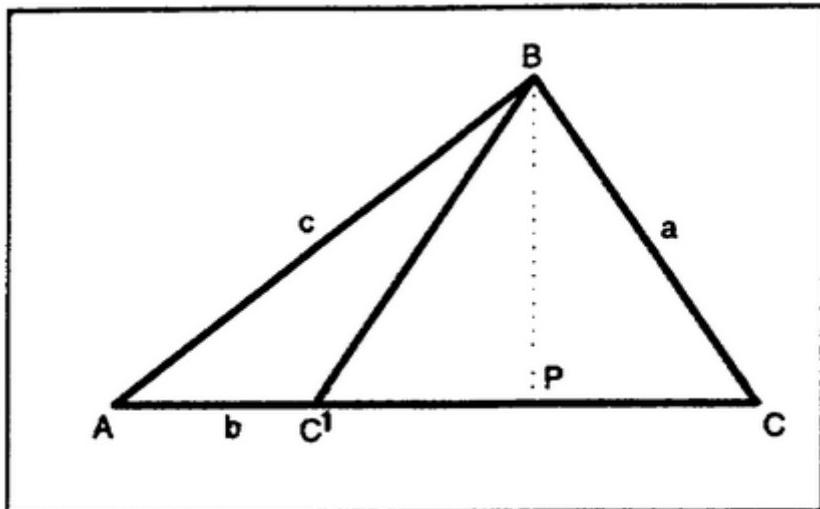


Figure 4-21. Case III

Solution:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (97.85)^2 + (106.66)^2 - (2)(97.85)(106.66)(\cos 73^\circ 19' 27'')$$

$$a^2 = 9574.62 + 11376.36 - 5989.63 = 14961.35$$

$$a = \sqrt{14961.35} = 122.32 \text{ feet (by natural functions)}$$

Now combine the different parts of the equation.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (9574.62 + 11376.36) - 5989.63 = 14961.35$$

When solving for the remaining two unknown angles of the triangle and two sides and the included angle are known, it is best to use the tangent law in the solution.

Example: In Figure 4-22, page 30, side c of the triangle equals 749.63 feet, side a equals 561.88 feet, and angle B equals $41^\circ 17' 32''$. Find angle A and angle C.

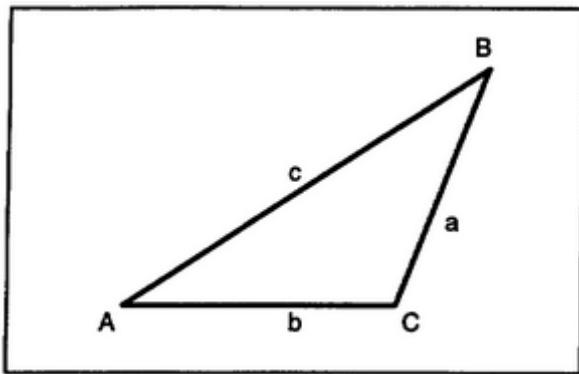


Figure 4-22. Solving for unknown angles (tangent law)

Solution: Find the value of angles A and C by subtracting angle B from 180°.

$$(C = A) = 180^\circ - 41^\circ 17' 32'' = 138^\circ 42' 28''$$

Then substitute the known values in the tangent equation.

$$\begin{aligned} \frac{c+a}{c-a} &= \frac{\tan \frac{1}{2}(C+A)}{\tan \frac{1}{2}(C-A)} \\ \tan \frac{1}{2}(C-A) &= \frac{c-a}{c+a} [\tan \frac{1}{2}(C+A)] \\ &= \frac{749.63 - 561.88}{749.63 + 561.88} (\tan 69^\circ 21' 14'') \\ \tan \frac{1}{2}(C-A) &= 0.379899 \\ \frac{1}{2}(C-A) &= 20^\circ 48' 12'' \end{aligned}$$

Now find the values of the unknown angles from the equations which follow:

$$\begin{aligned} C &= \frac{1}{2}(C+A) + \frac{1}{2}(C-A) \\ &= 69^\circ 21' 14'' + 20^\circ 48' 12'' = 90^\circ 09' 26'' \\ A &= \frac{1}{2}(C+A) - \frac{1}{2}(C-A) \\ &= 69^\circ 21' 14'' - 20^\circ 48' 12'' = 48^\circ 33' 02'' \end{aligned}$$

The above solution is by natural functions.

4-13. Case IV - Three Sides. When three sides of a triangle are known, it is best to solve for the remaining parts by the method of half tangents.

Example: In Figure 4-23, side a of the triangle equals 197.70, side b equals 206.15, and side c equals 184.42. Find the value of the three angles.

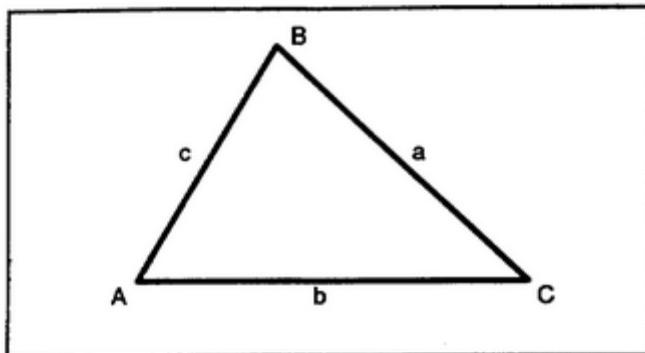


Figure 4-23. Case IV

Solution: Determine the value of s.

$$s = \frac{a + b + c}{2} = \frac{197.70 + 206.15 + 184.42}{2} = 294.135$$

Now solve for the angles.

$$\begin{aligned}\tan \frac{1}{2} B &= \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \\ &= \sqrt{\frac{(294.135 - 197.70)(294.135 - 184.42)}{(294.135)(294.135 - 206.15)}} \\ &= \sqrt{\frac{(96.435)(109.715)}{(294.135)(87.985)}} = \sqrt{\frac{10580.366}{25879.468}} \\ &= \sqrt{0.4088324} = 0.6394000\end{aligned}$$

$$\frac{1}{2} B = 32^\circ 35' 41.5''$$

$$B = 65^\circ 11' 23''$$

$$\begin{aligned}\tan \frac{1}{2} A &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\ &= \sqrt{\frac{(294.135 - 206.15)(294.135 - 184.42)}{(294.135)(294.135 - 197.70)}}\end{aligned}$$

$$= \sqrt{\frac{(87.985)(109.715)}{(294.135)(96.435)}} = \sqrt{\frac{9653.274}{28364.909}} \\ = \sqrt{0.3403225} = 0.5833734$$

$$\frac{1}{2} A = 30^\circ 15' 29.5''$$

$$A = 60^\circ 30' 59''$$

$$\tan \frac{1}{2} C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} \\ = \sqrt{\frac{(294.135 - 197.70)(294.135 - 206.15)}{(294.135)(294.135 - 184.42)}} \\ = \sqrt{\frac{(96.435)(87.985)}{(294.135)(109.715)}} = \sqrt{\frac{8484.833}{32271.022}} \\ = \sqrt{0.2629242} = 0.5127613$$

$$\frac{1}{2} C = 27^\circ 08' 49''$$

$$C = 54^\circ 17' 38''$$

Check your solution by adding angles A, B, and C to see if they equal 180° .

$$(60^\circ 30' 59'') + (65^\circ 11' 23'') + (54^\circ 17' 38'') = 180^\circ$$

The third angle could also be found by subtracting the first two angles from 180° . This, however, leaves no way to check the accuracy of the work performed.

LESSON 4

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. There is only one correct answer to each item. When you complete the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. Which of the following would be the result of subtracting $98^\circ 47' 52''$ from $36^\circ 40' 30''$?
 - A. $62^\circ 07' 22''$
 - B. $135^\circ 28' 43''$
 - C. $197^\circ 35' 44''$
 - D. $297^\circ 52' 38''$
2. If you divide an angle of $35^\circ 14' 15''$ into three equal segments, how many degrees, minutes, and seconds would each segment have?
 - A. $11^\circ 18' 45''$
 - B. $11^\circ 44' 45''$
 - C. $11^\circ 52' 45''$
 - D. $12^\circ 00' 45''$
3. If A and B are the acute angles of a right triangle, then cosine A is equal to _____.
 - A. $\cos B$
 - B. $\sin B$
 - C. $\sec B$
 - D. $\sec A$
4. In which of the four quadrants of a circle would an angle containing $282^\circ 16'$ be located?
 - A. IV
 - B. II
 - C. III
 - D. I
5. To solve a right triangle, two parts besides the right angle must be _____.
 - A. An obtuse angle
 - B. The hypotenuse
 - C. An exterior angle
 - D. A side

LESSON 4

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

<u>Item</u>	<u>Correct answer and feedback</u>
1.	D $297^\circ 52' 38''$ Accurately subtracting two ... (page 4-6, para 4-3e)
2.	B $11^\circ 44' 45''$ The original problem ... (page 4-2, para 4-2a)
3.	B $\sin B$ To differentiate between ... (page 4-8, para 4-4d)
4.	A IV Referring to Figure 4-11 ... (page 4-16, para 4-6b)
5.	D A side As you will recall ... (page 4-13, para 4-5h)

LESSON 5

SURVEYING EQUIPMENT

OVERVIEW

LESSON DESCRIPTION:

In this lesson, you will become familiar with surveying equipment.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will become familiar with the different types of equipment used in surveying operations.

CONDITION: You will be given the material contained in this lesson.

STANDARD: You will correctly answer the practice exercise questions at the end of this lesson.

REFERENCES: The material contained in this lesson was derived from TM 5-232, FM 5-233, NAVEDTRA 10696, and CDC 3E551A.

INTRODUCTION

Accuracy in surveying is essential because other engineering factors that are involved after the survey is complete depend on the surveying results. Construction surveys deal with determining the relative positions of points on the earth's surface. These points are used to locate and lay out roads, airfields, and man-made structures (such as buildings, sewer lines, utility lines, and any type of proposed or existing structure). Construction surveys also identify terrain features that are used to draw large-scale maps. No matter what type of survey is needed, some type of surveying equipment will be required. Each surveying operation requires certain specific types of equipment. These surveys can vary in many ways, depending on the technical requirements needed. This lesson will familiarize you with the surveying equipment that you will encounter in the field.

PART A - UNIVERSAL SURVEYING INSTRUMENTS

5-1. Tools and Devices. Primarily, the surveying instruments that you will be using are precise tools with which measurements are made. Many of the instruments have similar features. These instruments include

- Tripods to hold the instrument steady at a convenient height.

- Level vials and leveling thumb screws to make the telescope level and parallel to a horizontal line of sight.
- Optical devices, such as crosshairs in a telescope to sight on a target.
- Magnification lenses on the telescope to magnify a target.
- Plumb bobs or optical plummets to align the instrument exactly over a selected point.

The surveying instruments that you will become familiar with are the one-minute theodolite, geodimeter (total station), and levels. These items of equipment are needed to help you determine the angles and elevations that are required for a construction survey.

a. One-Minute Theodolite. The one-minute theodolite is used to obtain both horizontal and vertical angles. The theodolite is a compact, lightweight, dustproof optical-reading, directional-type instrument (see Figure 5-1). It may also be used as a repeating-type instrument for measuring horizontal angles by using the repeating damp. The scales are readable directly to the nearest minute and may be illuminated by sunlight or artificial light. The surveying points are observed through the instrument and the angles read through an optical microscope on horizontal and vertical scales inside the instrument. There are two versions of this theodolite—the engineer version, which reads directly to one minute (more commonly known as the T16) and the artillery version, which reads directly to 0.2 mil (more commonly known as the T2).

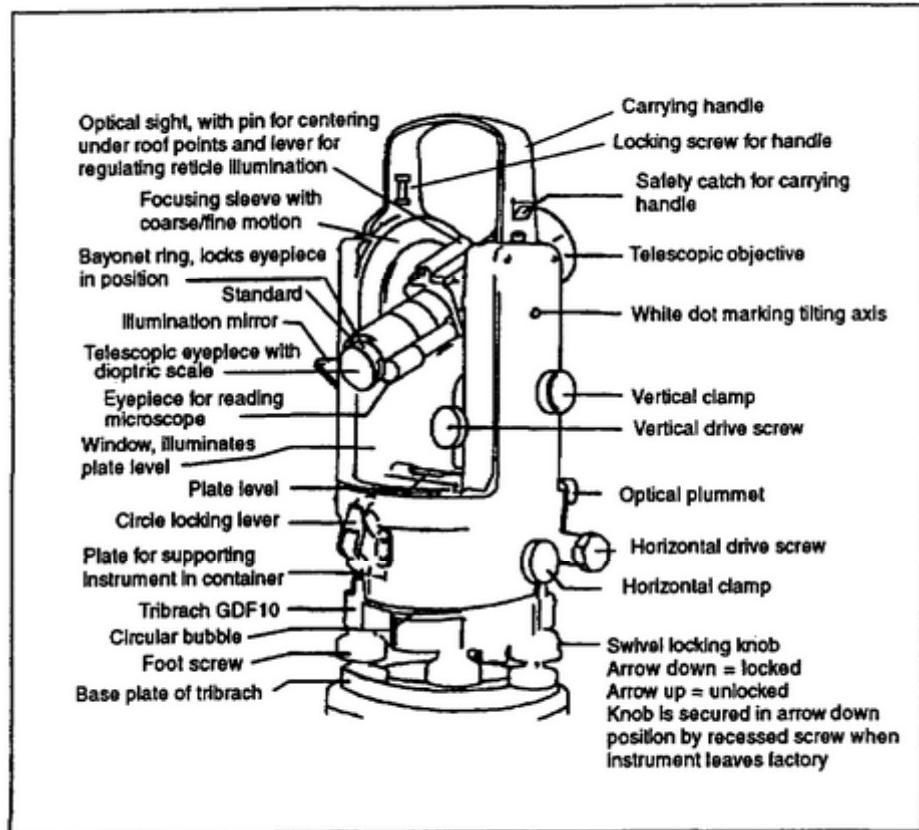


Figure 5-1. One-minute theodolite

b. Geodimeter. The geodimeter is an automated, integrated surveying instrument (AISI) commonly referred to as a total station (see Figure 5-2, page 5-4). It combines a theodolite with an electronic distance-measurement (EDM) instrument and an electronic data collector. The geodimeter can measure angles up to one-second accuracy, distances (up to 3 kilometers) to the thousandths, and elevations to the hundredths of a foot or meter. It has the capacity to store up to 6,000 points of surveying information. It can operate up to eight hours continuously on the batteries provided, or a 12-volt car battery can be used. The geodimeter comes with factory-set programs designed for standard surveying operations and can be programmed for special surveying operations. The geodimeter is used in conjunction with the civil software program Terramodel. This software accepts the downloaded information from the geodimeter and allows rapid development of the surveying site by producing civil engineering drawings.



Figure 5-2. The geodimeter (AISI)

c. Levels. Discussed in the following paragraphs are the different types of levels that may be used in construction surveys.

(1) Dumpy Level. The dumpy level is a very sturdy and reliable instrument that was used extensively for leveling operations until it was replaced by more modern equipment (see Figure 5-3). Its sighting device is a 28 variable-power telescope with a maximum length of 18 inches and an erecting eyepiece that changes the inverted/upside-down image so that it can be seen right-side up. The focusing knob is normally external. Rotating the focusing knob brings the target into clear focus. The reticle has two crosshairs at right angles to each other, and some models have stadia hairs for distance measurement to the nearest foot or meter. Rotating the eyepiece brings the crosshairs into focus. The telescope and level-bar assembly are mounted on a spindle that permits the unit to be moved only in a horizontal plane. The telescope and level bar cannot be elevated or depressed. The dumpy level's telescope is rigidly attached to the level bar that holds an adjustable, highly sensitive level vial. The azimuth clamp and azimuth tangent screw allow for slow motion of the telescope for accurate centering on a target. The spindle mounts in a four-screw leveling head that rests on a footplate. The footplate screws onto the threads of the tripod. When the instrument is properly leveled and adjusted, the horizontal line of site, which is defined by the horizontal crosshair, forms a horizontal plane.

(2) Automatic Level. The automatic level (also called the auto level) is a self leveling level that has become the most popular, standard-type level used in construction surveys (see Figure 5-4). It is very easy to use and can be set up quickly. The automatic level has upgraded surveying operations by taking the place of the dumpy level, which requires a great amount of time to center its bubble and reset its position. The automatic level has a small circular level called a bull's-eye level and only three leveling screws. The leveling screws are on a triangular footplate and are used to center the bubble in the bulls-eye level. The line of sight automatically becomes horizontal and remains horizontal as long as the bubble stays centered. Inside the automatic level, a gravity-suspended prism (called a compensator) is hung on fine, nonmagnetic wires. The action of gravity on the compensator causes the

prism to swing freely into a position so that a horizontal line of sight is achieved. This horizontal line of sight is maintained even when the telescope is unlevel or the instrument is disturbed because the prism swings freely with gravity.

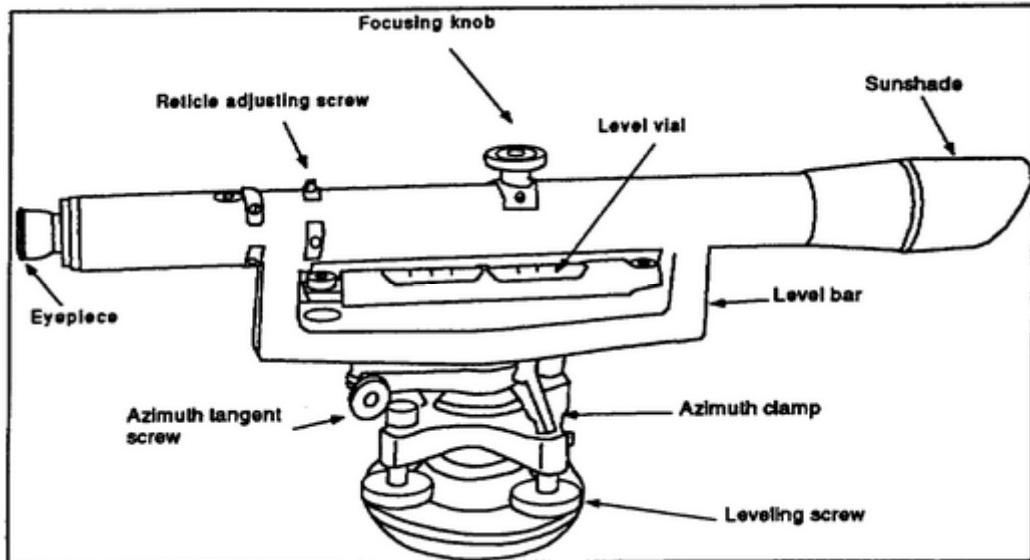


Figure 5-3. The dumpy level

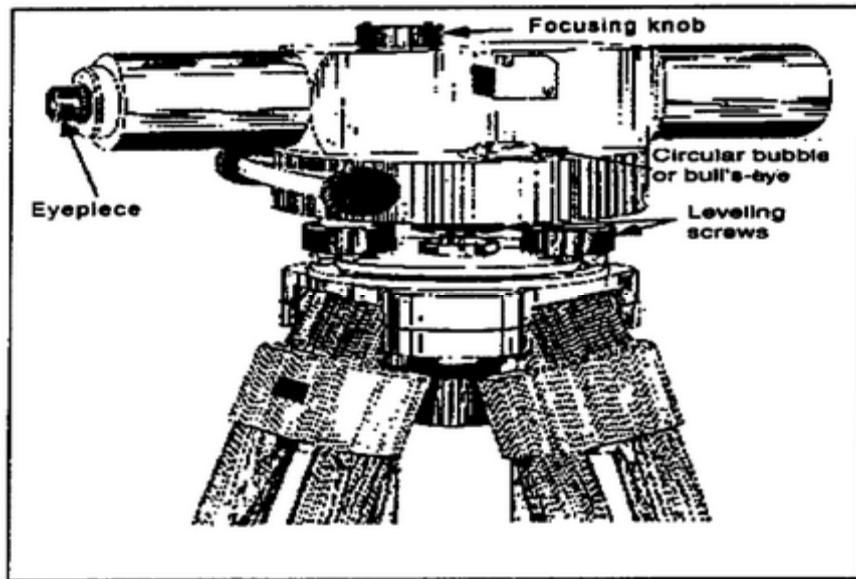


Figure 5-4. The automatic level

(3) Hand Levels. The hand level, like all surveying levels, is an instrument that combines a level vial and a sighting device (see Figure 5-5, page 5-6). It is a very simplistic device that is used for taping and for rough determinations of elevations. The basic principle is that if the bubble is centered while sighting through the tube, the line of sight is horizontal.

(a) For example, in cross-sectional work, terrain irregularities may cause elevations to go beyond the automatic level's range from a setup. A hand level is useful for extending approximate elevations off the control-survey line beyond the limits of the automatic level. For greater stability, the hand level may be rested against a tree, a Philadelphia rod, or a range pole.

(b) On a hand level, a level vial is mounted atop a slot in the sight tube in which a reflector is set on a 45° mirror. This allows the observer, while sighting through the tube, to see the landscape or object, the position of the bubble in the vial, and the index line at the same time. The hand level has no magnification capabilities, therefore, the distances sighted are relatively short.

(c) In Figure 5-5, view A shows a Locke hand level that is very simplistic in design and view B shows an Abney hand level (or clinometer) which has a reversible, graduated arc assembly mounted on one side and may be used for measuring vertical angles and percent of slope. The lower side of the arc is graduated in degrees, and the upper side is graduated in percent of slope. The level vial of the Abney hand level is attached at the axis of rotation of the index arm. When the index arm is set to zero, the instrument is used like a Locke hand level. When it is used as a clinometer, the object is sighted and the level tube is rotated about the axis of rotation until the bubble is centered. The difference between the line of sight and the level-bubble axis can be read in degrees or percent of slope from the position of the index arm.

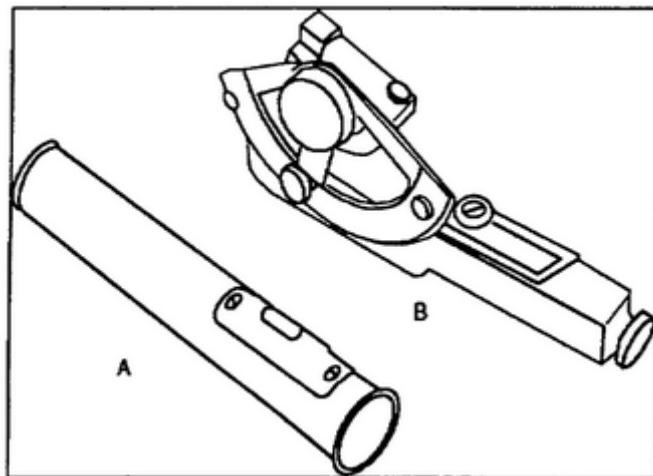


Figure 5-5. Hand levels

PART B - FIELD EQUIPMENT

5-2. Hand Tools. The term field equipment, as used in this lesson, includes all devices, tools, and instrument accessories used in connection with field measurements.

a. When conducting a survey across rough terrain, various types of tools will be needed to clear the line, that is, cut down brush and other natural growth as needed (see Figure 5-6). Surveying procedures usually permit the bypassing of large trees. However, it may be necessary to fell a tree. If heavy equipment is working in the area, it may be used to fell the tree; if not, a chain saw may be used. If a chain saw is not available, use an ax.

The machete and brush hook may be used to clear small saplings, bushes, or similar growth. Axes and hatchets are used to mark trees by blazing and may also be used to fell trees.

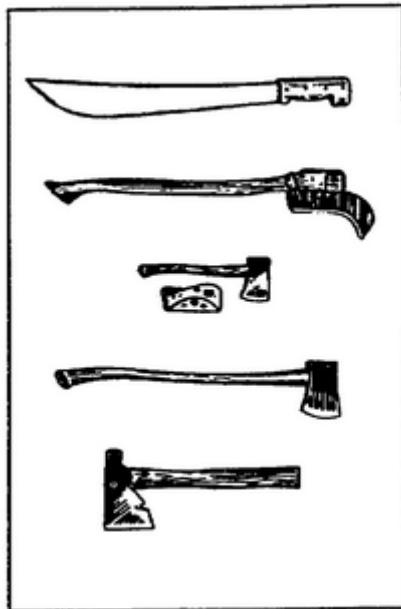


Figure 5-6. Cutting tools

b. A hatchet or single-bit ax is used to drive hubs, stakes, pipe, and other markers into the ground. A sledgehammer, however, is a more suitable tool for this purpose. A double-faced, long-handled sledgehammer is shown in Figure 5-7, page 5-8. It is swung with both hands. There are also short-handled sledgehammers that can be swung with one hand. A sledgehammer is classified according to the weight of the head. Common weights are 6, 8, 10, 12, 14, and 16 pounds. The 8- and 10- pound weights are the most commonly used.

c. When searching for hidden markers, you may need a shovel or a pick like the ones shown in Figure 5-7, page 5-8, to clear off the topsoil. In soft ground, such as loose, sandy soil, you may prefer to use a square-headed shovel or a probing steel rod to locate buried markers.

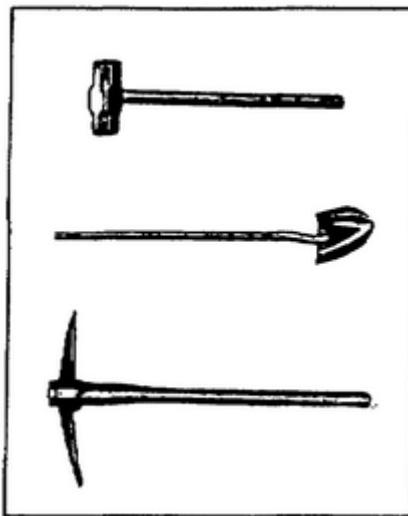


Figure 5-7. Sledgehammer, shovel, and pick

PART C - ASSOCIATED SURVEYING EQUIPMENT

5-3. Surveying Tapes. Tapes are used in surveying to measure horizontal, vertical, and slope distances. They may be made of a ribbon or band of steel, an alloy of steel, cloth reinforced with metal, or synthetic materials. Tapes are issued in various lengths and widths and graduated in a variety of ways. Various types of surveying tapes are shown in Figure 5-8. View A shows a nonmetallic tape, view B shows a steel tape on an open reel, view C shows a metallic tape on a closed reel, and View D shows a special type of low-expansion steel tape (called an Invar tape or a Lovar tape) used for geodetic work and for checking the length of regular steel tapes. The Invar and Lovar tapes are very precise and will not react to temperature changes.

a. Metallic Tape. A metallic tape is made of high-grade synthetic material with strong wire strands (bronze, brass, and copper) woven in the warped face of the tape and coated with a tough, water-resistant plastic for durability. Standard lengths are 50 and 100 feet. Most metallic tapes are graduated in feet and decimals of feet, but some are graduated in feet and inches to the nearest 1/4 inch, meters, and centimeters. Metallic tapes are generally used for rough measurements, such as cross-sectional work, roadwork slope staking, and side shots in topographic surveys.

b. Nonmetallic Tapes. Some surveyors prefer to use nonmetallic tapes that are woven from synthetic yarn, such as nylon, and coated with plastic. Nonmetallic tapes are similar to metallic tapes in their use, lengths, and graduations.

c. Steel Tapes. For direct linear measurements of ordinary or more accurate precision, a steel tape is required. The most commonly used length is 100 feet, but tapes are also available in 50-, 200-, 300-, and 500-foot lengths. All tapes except the 500-foot tape are band-type; the common band widths are 1/4 and 5/16 inch. The 500-foot tape is usually a flat-wire type. Metric steel tapes are available and are commonly used overseas, with the most common lengths being 30 and 50 meters. Most steel tapes

are graduated in feet and decimals of feet, but some are graduated in feet and inches, meters, or other linear units. Steel tapes are sometimes equipped with a reel on which they can be wound. These tapes can be, and often are, detached from the reel for more convenient use in taping.

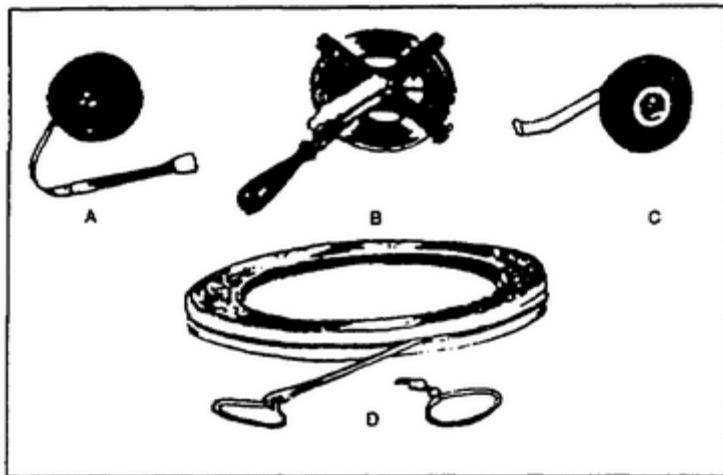


Figure 5-8. Surveying tapes

5-4. Surveying Accessories. Surveying accessories include the equipment, tools, and other devices used in surveying that are not considered to be an integral part of the surveying instrument itself. For example, when you run a traverse, your primary instruments may be the theodolite and the steel tape. The accessories you will need to do the actual measurement are-

- A tripod to support the theodolite.
- A range pole to sight on.
- A plumb bob to center the instrument on the point.
- Tape supports to support the tape from sagging if the survey is of high precision.

It is important that you become familiar with the proper care of this equipment and use it properly.

a. Tripod. The tripod is the base or foundation that supports the surveying instrument and keeps it stable during observations. A tripod consists of a head to which the instrument is attached, three wooden or metal legs that are hinged at the head, and pointed metal shoes on each leg to be pressed or anchored into the ground to achieve a firm setup (see Figure 5-9, page 5-10). The leg hinge is adjusted so that the leg will just begin to fall slowly when it is raised to an angle of about 45° . The tripod head may have screw threads on which the instrument is mounted directly, a screw protecting upward through the plate, or a hole or slot through which a special bolt is inserted to attach to the instrument. When mounting the instrument on the tripod, firmly grip it to avoid dropping it. Hold the theodolite by the

right standard (opposite the vertical circle) while you are attaching it. The automatic and dumpy levels should be held at the center of the telescope. Both theodolites and levels should be gripped near the base of the instrument with the opposite hand. The instrument should be screwed down to a firm bearing but not so tightly that it will bind or the screw threads will strip.

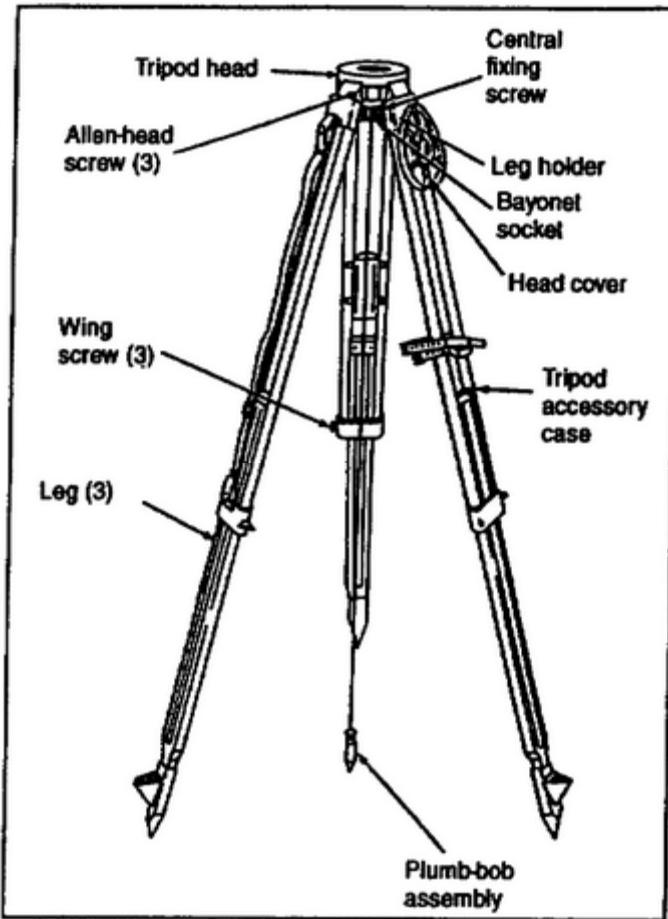


Figure 5-9. Tripod detail

(1) **Types.** There are two types of tripods that surveyors use: the fixed leg and the extension leg (see Figure 5-10). These tripods can also have wide frames like those shown in Figure 5-11, which have greater torsional stability and tend to vibrate less in the wind.

(a) **Fixed Leg.** The fixed-leg tripod is also called a stilt-leg or rigid tripod. Each fixed leg may consist of two lengths of wood as a unit or a single length of wood split at the top, which is attached to a hinged tripod head fitting and to a metal shoe. At points along the length of the leg, perpendicular brace pieces are sometimes added to give greater stability. The fixed legs must be swung in or out in varying amounts to level the head. Instrument height is not easily controlled, and the observer must learn the correct spread of the legs to get the desired height.

(b) **Extension Leg.** The extension-leg tripod is also called a jack-leg tripod. Each extension leg is made of two sections that slide longitudinally. On rough ground, the legs are adjusted to different lengths to establish a horizontal tripod head or to set the instrument at the most comfortable

working height for the observer. A leg may be shortened and set as shown in the extreme left view of Figure 5-10.

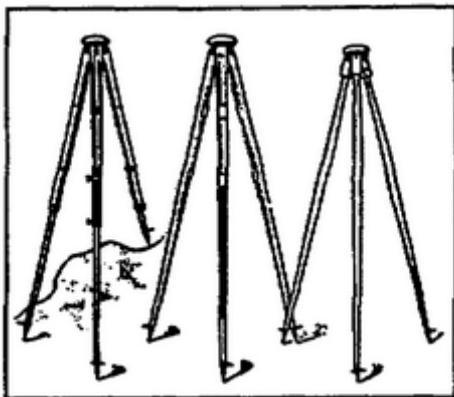


Figure 5-10. Fixed- and extension-leg tripods

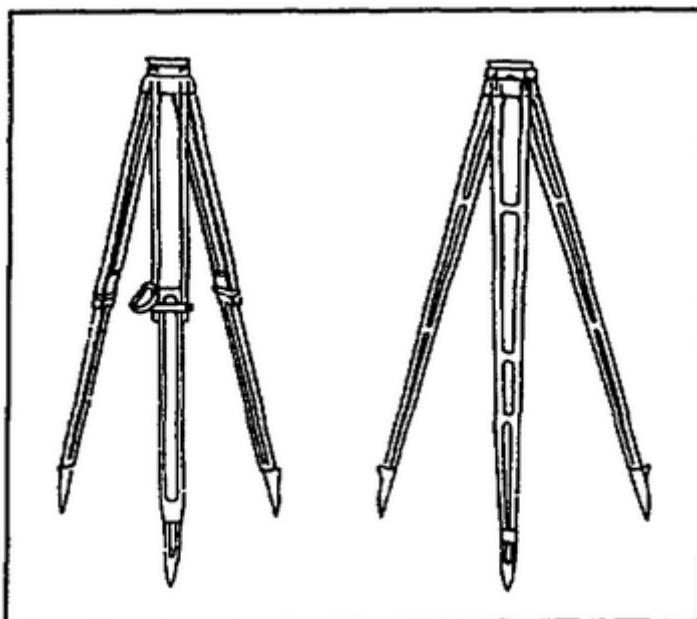


Figure 5-11. Wide-frame tripods

(2) Setup. When setting up a tripod, you should be sure to place the legs so that you achieve a stable setup. First, loosen the restraining strap from around the three legs and secure it around one leg. While standing over the setup mark, grip the tripod with two of the legs close to the body and, by using one hand, push the third leg out away from the body until it is about 3 feet from the mark. Lower the tripod until the third leg is on the ground. Place one hand on each of the first two legs and spread them while taking a short backward step, using the third leg as a pivot point. When the two legs look about as far away from the mark as the third one and all three are about equally spaced, lower the two legs and press them into the ground. Make any slight adjustment to level the head further by moving the third leg a few inches in or out before pressing it into the ground.

(a) On smooth or slippery surfaces, you should tighten the tripod's leg hinges while setting it up to prevent the legs from spreading and causing the tripod to fall. If there are holes or cracks in the ground, use them to brace the tripod. Sometimes, as a safety factor, you should tie the three legs together or brace them with rocks or bushes after they are set to keep them from spreading. If setups are to be made on a slippery finished floor, rubber shoes may be fitted to the metal shoes or an equilateral triangle leg retainer may be used to prevent the legs from sliding.

(b) When you are setting up on sloping ground, place the third leg uphill and at a greater distance from the mark. Set the other two legs as before, but before releasing them, make sure that the weight of the instrument and the tripod head does not overbalance the tripod and cause it to slip or fall.

(c) Proper care must be observed in handling the tripod. When the legs are set in the ground, apply pressure longitudinally. Pressure across the leg can crack the wooden pieces. The hinge joint should be adjusted and not over tightened to the degree that it would cause strain on the joint or strip or lock the metal threads. The tripod head should be kept covered with the head cover or protective cap when not in use, and the head should not be scratched or burred by mishandling. When the tripod is in use, the protective cap is to be placed in the instrument box to prevent it from being misplaced or damaged. Any damage to the protective cap can be transferred to the tripod head. Mud, clay, or sand adhering to the tripod must be removed, and the tripod should be wiped with a damp cloth and dried. The metal parts should be coated with a light film of oil or wiped with an oily cloth. Foreign matter can get into hinged joints or on smooth surfaces and cause wear. Stability is the tripod's greatest asset. Instability, wear, or damaged bearing surfaces on the tripod can evolve into unexplainable errors in the final surveying results.

b. Range Poles. A range pole is a wood, fiberglass, or metal pole, usually about 8 feet long and about 1/2 to 1 inch in diameter. It has a steel-weighted point and is painted in alternate bands of red and white to increase its visibility. The bands are 1 foot long and can be used as a rough measurement guide using stadia estimation. Figure 5-12 shows a variety of range poles. The range pole is held vertically on a point or plumbed over a point so that the point may be observed through an optical instrument. It is primarily used as a sighting rod for either linear or angular measurements. For work of ordinary precision, chainmen may stay on line by observing a range pole.

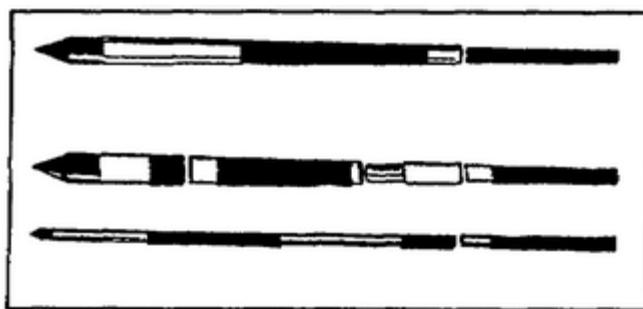


Figure 5-12. Range poles

c. Plumb Bob, Cord, and Target. A plumb bob is a pointed, tapered brass or bronze weight that is suspended from a cord to determine the plumb line from a point on the ground. Common weights for

the plumb bobs are 6, 8, 10, 12, 14, 16, 18, and 24 ounce; the 12- and 16-ounce weights are the most popular. Typical plumb bobs are shown in Figure 5-13.

(1) A plumb bob is a precise instrument and must be cared for as such. If the tip becomes bent, the cord from which the bob is suspended will not occupy the true plumb line over the point indicated by the tip. A plumb bob usually has a detachable tip, as shown in Figure 5-13. If the tip becomes damaged, it can be renewed without replacing the entire instrument.

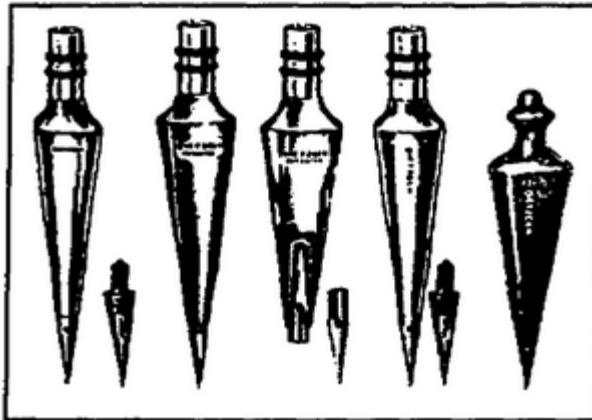


Figure 5-13. Plumb bobs

(2) Each survey party member should be equipped with a leather sheath for the plumb bob, and it should be placed in the sheath whenever it is not in use. The cord from a plumb bob can be made more conspicuous for observation purposes by attaching an oval-shaped aluminum target (Figure 5-14, view A, page 5-14). The oval-shaped target has reinforced edges with alternate red and white quadrants on its face. Also, a flat rectangular plastic target may be used (Figure 5-14, view B, page 5-14). It has rounded corners with alternate red and white quadrants on its face. These plumb-bob string targets are pocket size (approximately 2 by 4 inch).

d. Optical Plummet. The optical plummet is a device built into the theodolite or the tribrach of some instruments to center the instrument over a point. Its working principle is shown in Figure 5-15, page 5-14.

The plummet consists of a small prismatic telescope with a crosshair or marked-circle reticle adjusted to be in line with the vertical axis of the instrument. After the instrument is leveled, a sighting through the plummet will check the centering over a point quickly. The advantages of the plummet over the plumb bob are that it permits the observer to center over a point from the height of the instrument stand and that it is not affected by the wind. A plumb bob requires someone at ground level to steady it and to inform the observer on the platform how to move the instrument and when it is exactly over the point. With the plummet, the centering and checking is done by the observer.

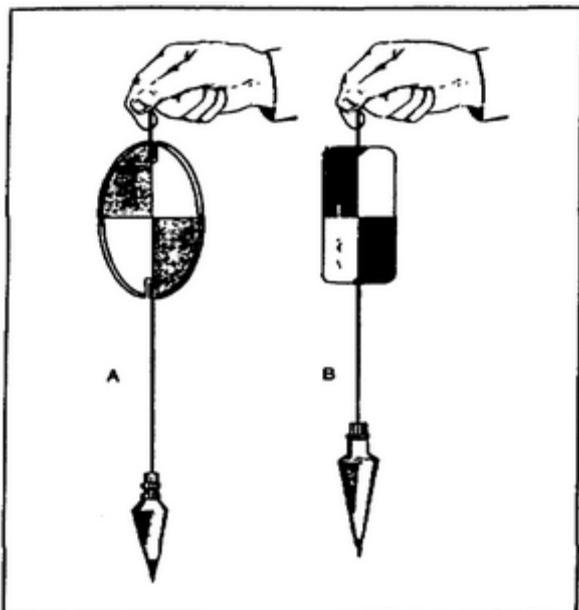


Figure 5-14. Plumb bob, cord, and target

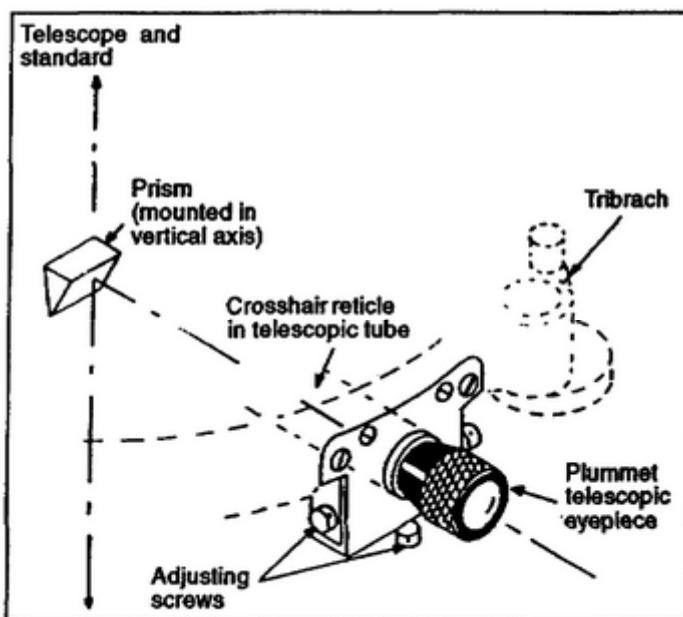


Figure 5-15. Optical plummet

e. Tape Clamp. There is usually a leather thong at each end of a tape that can be held when the full length of the tape is used. When only part of the tape is used, the zero end can be held by the thong, and the tape can be held at an intermediate point by means of a tape clamp (see Figure 5-16). These scissor-type clamps grip the tape tightly without bending or damaging it. The tape clamps are especially helpful when the tape needs to be pulled tightly. They make the tape easy to grip during measurements.

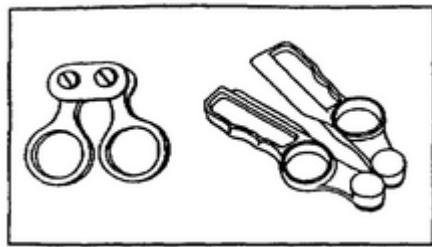


Figure 5-16. Tape clamps

f. Tension Scale. When a steel tape is held above ground between two crew members without support throughout, a certain amount of tension must be applied to reduce the sag in the tape. This can be done by using a tension scale, which is graduated in pounds from 0 to 30 (see Figure 5-17). It is clipped to the eye at the end of the tape, and tension is applied until the desired reading appears on the scale. The proper amount of continuous tension that needs to be applied to a steel tape is 20 pounds.

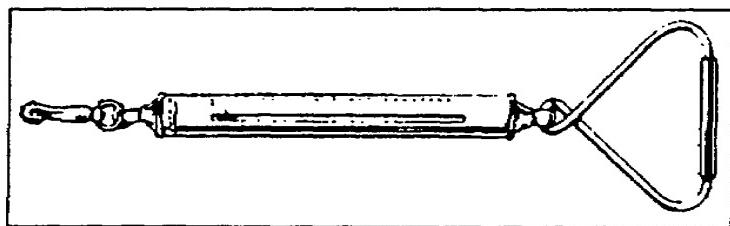


Figure 5-17. Tension scale

g. Taping Pin. A taping pin is a metal pin that is 1 foot long. It has a circular eye at one end and a point for pushing it into the ground at the other (see Figure 5-18, page 5-16). These pins come in sets of 11 and are carried on a wire ring that is passed through the eyes of the pins. Taping pins can be used to temporarily mark points in a great variety of situations. They are also used to keep count of tape increments in the taping of long distances. Each pin represents one tape length measured but not necessarily a full tape length. The number of taping pins used equals the number of distances that are measured and recorded in the recording book.

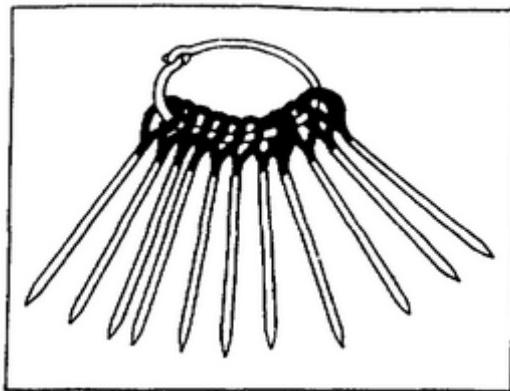


Figure 5-18. Taping pins

h. Leveling Rods. A leveling rod is a wooden rod that is used to measure the vertical distance above a point. This point may be a permanent elevation (bench mark), a temporary bench mark (a turning pin or stake), a man-made object, a constructed surface, or a natural point on the ground surface.

(1) Use. The leveling rod may be read directly by the instrument man sighting through the telescope, or it may be target read. Conditions that hinder direct reading, such as poor visibility, long sights, and partially obstructed sights (through brush or leaves), sometimes make it necessary to use a target. A target is also used to mark a rod reading when numerous points are set to the same elevation or a certain constant grade is needed from one instrument setup. In Figure 5-19, view B shows a rod with metric measurements; the graduations of the rod are in meters, decimeters, and centimeters. The targets that are furnished with the metric rod have a vernier that permits reading the scale to the nearest millimeter. The metric rod can be extended from 2.0 to 3.7 meters.

(2) Types. The different types of leveling rods are discussed in the following paragraphs.

(a) Philadelphia Rod. The most popular of all is the Philadelphia rod, which is a graduated two-section wooden rod (see Figure 5-19, view A). It can be extended from 7 to 13 feet and each foot is subdivided into hundredths of a foot. Instead of each hundredth being marked with a line or tick, the distance between alternate ones is painted black on a white background. Thus, the value for each hundredth is the distance between the colors; the top of the black increment is even values and the bottom of the black increment is odd values. The tenths are numbered in black and the feet are numbered in red. This rod may be used with the level, theodolite, and hand levels on occasion to measure the difference in elevation or it may be used for topographic land surveys.

Targets for the Philadelphia rod are usually oval, with the long axis at right angles to the rod and the quadrants of the target painted alternately red and white. The target is held in place on the rod by a C-clamp and a thumbscrew. A lever on the face of the target is used for fine adjustment of the target to the line of sight of the level. The targets have rectangular openings about the width of the rod and 0.15 feet high through which the face of the rod may be seen. A linear vernier scale is mounted on the edge of the opening with the zero on the horizontal line of the target for reading to thousandths of a foot. When the

target is used, the rodman takes the rod reading.

(b) San Francisco Rod. The San Francisco rod is used for direct reading only and is available with three sliding sections.

(c) Chicago Rod. The Chicago rod is available with three or four sections that, instead of sliding, are joined at the end to each other like a fishing rod.

(d) Lenker Rod. The Lenker rod is a two-section rod similar to the Philadelphia but is graduated in feet and inches to the nearest one-eighth inch rather than the decimal. The upper section of the Lenker rod has the graduations on a continuous metal belt that can be rotated to set any desired graduation at the level of the height of the instrument (HI). To use the rod, set it on the bench mark and bring the graduation that indicates the elevation of the bench mark level with the HI. As long as the level remains at that same setup whenever you set the rod on a point, you can read the elevation of the point directly. In short, the Lenker rod does away with the necessity for computing the elevations.

(e) Lovar Rod. The Lovar rod is a high-precision leveling rod. It is usually T-shaped in cross section and has the scale inscribed on the metal strip. High-precision leveling rods usually have tapering, hardened-steel bases and some are equipped with thermometers so that the temperature correction can be applied. These rods generally contain built-in rod levels.

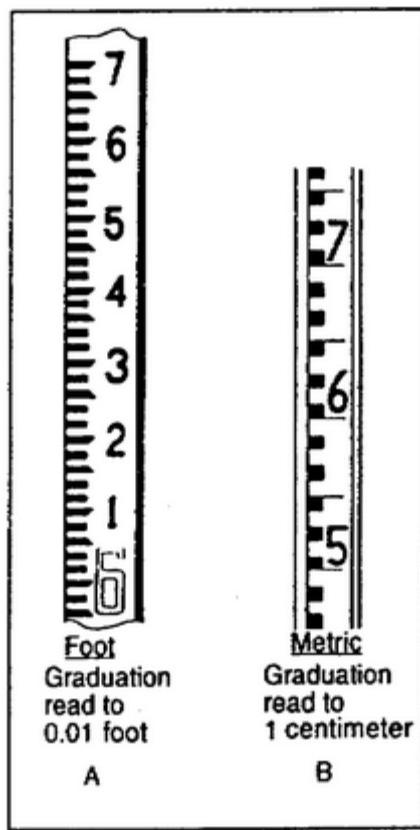


Figure 5-19. Leveling rods

(3) Care. Proper care should be taken of the leveling rods, such as keeping them clean (free of sand and dirt), straight, and readable. Leveling rods must be carried over the shoulder or under the arm from point to point. They must be collapsed in their original configuration when carrying them over long distances or when transporting them. Dragging them through the brush or along the ground will wear away or chip the paint. Do not set the rod with the numbers face down, as this will wear off the painted numbers. When not in use, the leveling rods should be stored in their cases, when available, to prevent warping. The cases are generally designed to support the rods either flat or on their sides. The rods should not be leaned against a wall or placed on the damp ground for any extended period, since this can produce a curvature in the rods and result in leveling errors.

i. Rod Levels. When a rod reading is made, it is accurate only if the rod is perfectly plumbed. If the rod is out of plumb, the reading will be greater than the actual vertical distance between the HI and the base of the rod. Therefore, to ensure a truly plumbed leveling rod, a rod level should be used. The two types of rod levels that are generally used with standard leveling rods are shown in Figure 5-20. The one on the left is called the bull's-eye level, and the one on the right is the vial level. Figure 5-21 shows the proper way of attaching the bull's-eye level; the vial level is attached in the same manner.

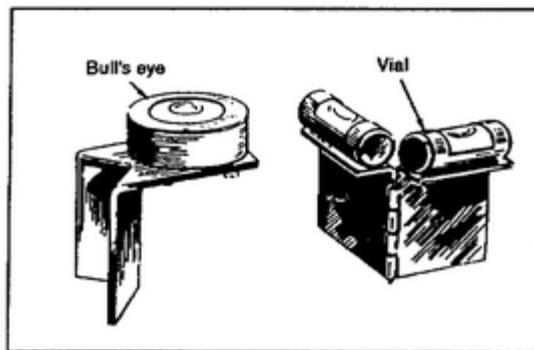


Figure 5-20. Rod levels

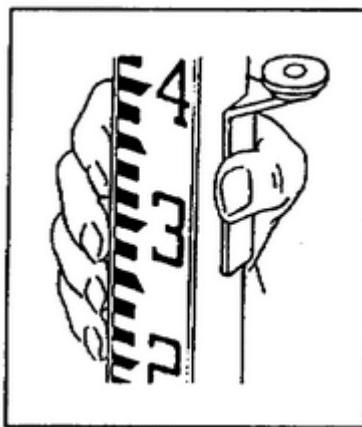


Figure 5-21. Proper attachment of rod level

j. Stadia Boards. In determining linear distance by stadia, you observe a stadia rod or stadia board through a telescope containing stadia hairs and note the size of the interval intercepted by the hairs. A typical stadia board is shown in Figure 5-22. Note that it is graduated in a manner that facilitates counting the number of graduations intercepted between the hairs. Each tenth of a foot is marked by the point of one of the black, saw-toothed graduations. The interval between the point of a black tooth and the next adjacent white gullet between two black teeth represents 0.05 foot.

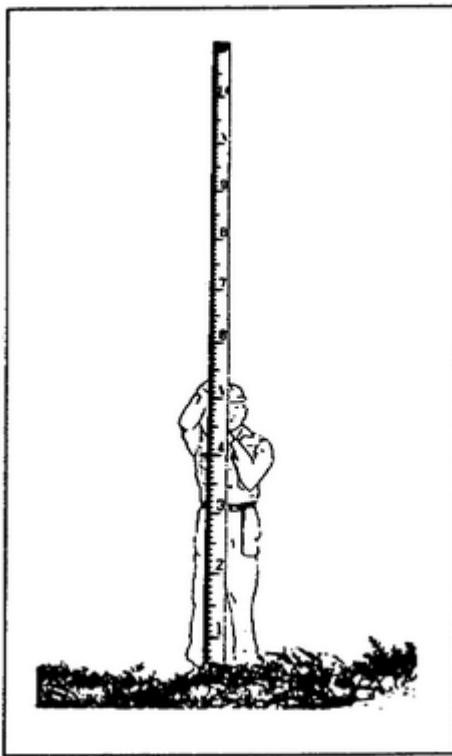


Figure 5-22. Stadia boards

k. Adjusting Pins. Surveying instruments are built in such a way that minor adjustments can be performed in the field without much time loss. The adjustments are made by loosening or tightening the capstan screws with adjusting pins. These pins are included in the instrument box. They come in various sizes, depending on the type of instrument and the hole sizes of its capstan screws. To avoid damage to the head of the capstan screw, use the correct size adjusting pin.

If you lose or break a pin, surveying-equipment dealers will usually replace them free of charge. While conducting a survey, the adjusting pins should be carried in your pocket. This will save valuable time when the pins are needed. Do not use wires, nails, screwdrivers, ink pens, or similar pointed items as substitutes for adjusting pins.

l. Tape Repair Kit. Even though you handle the tape properly and carefully during field measurements, some tapes still break under unforeseen circumstances. During taping operations in the field, the surveyor should always be sure to have an extra tape or a tape repair kit with him so that he

can rejoin any broken tape (Figure 5-23). The tape repair kit usually contains a pair of small snips, the tape sections of proper size and graduations, a hand punch or bench punch with block, an assortment of small rivets, a pair of tweezers, a small hammer, and a small file. Before reusing a repaired tape, always check its accuracy by comparing it with another tape that you know is correct.

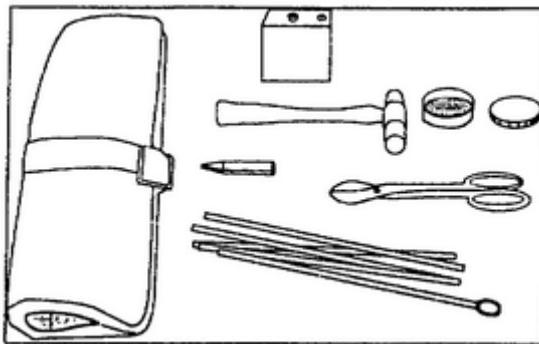


Figure 5-23. Tape repair kit

5-5. Field Supplies. Field supplies consist of a variety of materials used to mark the locations of points in the field. These materials are discussed in the following paragraphs.

a. Surveying Markers. The material used as a survey-point marker depends on where the point is located and whether the marker is to be of a temporary, semipermanent, or permanent character.

(1) Temporary Markers. For purely temporary marking, it is often unnecessary to expend any marking materials. For example, a point in ordinary soil is often temporarily marked by a hole made with the point of a plumb bob, a taping pin, or some other pointed device. In rough taping of distances, even the mere imprint of a heel in the ground may suffice. A point on a concrete surface may be temporarily marked by an X drawn with keel (lumber crayon), a pencil, or some similar marking device. A large nail serves well as a temporary point in relatively stable ground or compacted materials.

(2) Semipermanent Markers. Wooden hubs and stakes are extensively used as semipermanent markers of points in the field. The principal distinction between the two markers is that the top of the hub is usually driven flush, or almost flush, with the ground; whereas with the stake, it is left above the ground several inches.

(a) Wooden hubs are used to mark the station point for an instrument setup. A survey's tack, made of galvanized iron or stainless steel with a depression in the center of the head, is driven into the top of the hub to locate the exact point where the instrument is to be plumbed. Wooden hubs are usually made of a 2- by 2-inch stock and are from 4 to 12 inches long. The average length is about 8 inches; however, shorter lengths can be used in hard ground and longer lengths can be used in soft ground.

(b) Wooden stakes that are improvised in the field may be cylindrical or any other shape that is available. However, manufactured stakes are rectangular in cross section because the faces of the

stake are often inscribed with data relevant to the point that the stake is marking. A stake that marks a bench mark, for instance, is inscribed with the symbol that identifies the bench mark and the elevation. A stake that marks a station on a traverse is inscribed with the symbol of the particular station, such as 2 + 45.06. A grade stake is inscribed with the number of vertical feet of cut (material to be excavated) or fill (material to be filled in) required to bring the elevation of the surface to the specified grade elevation. Figure 5-24 shows typical dimensions for an average-sized hub and stake. These dimensions, however, may be modified as situations arise, such as material limitations.

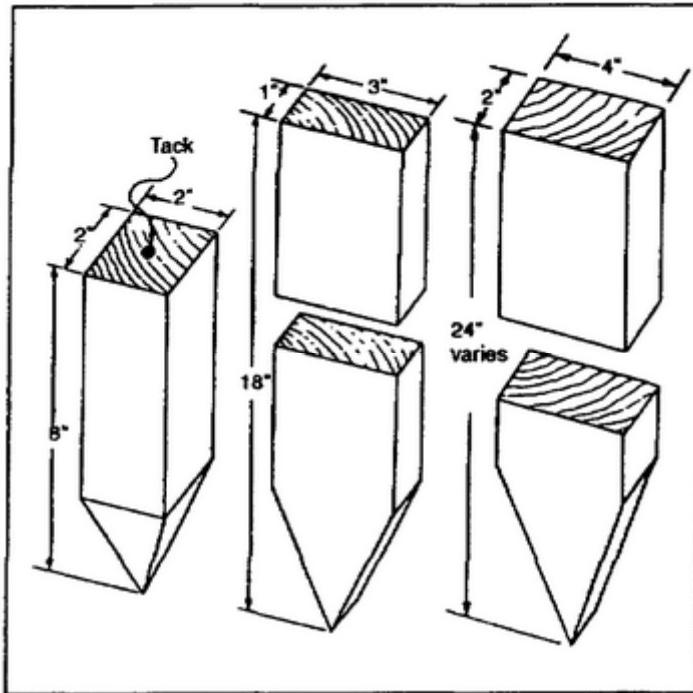


Figure 5-24. Hubs and stakes

(3) Permanent Markers. Permanent markers are used to mark points that are to be used for a long period of time. All permanent markers should be referenced so that they can be replaced if disturbed. Horizontal and vertical control stations are generally marked with permanent markers. These markers could be in the following forms:

(a) Surveyors Tacks, Spikes, and Nails. They are often driven into growing trees, bituminous, or other semisolid surfaces as permanent markers. A nail will be more conspicuous if it is driven through a bottle cap, a washer, a plastic tape, or a "shiner." A shiner is a thin metal disk much like the top or bottom of a frozen fruit-juice can.

(b) Spad. It is a nail equipped with a hook for suspending a plumb bob. It is driven into an overhead surface, such as the top of a tunnel. The suspended plumb bob indicates the point on the floor that is vertically below the spad.

(c) Crosscuts on Existing Concrete Structures or Rock Outcrops. Points on concrete or stone surfaces are often marked with an X by using a hammer and chisel. Another way to do this is to cut holes with a star drill and then plug them with lead.

(d) Metal Pipe. Metal pipe (usually called iron pipe regardless of the actual metal used) runs in lengths of about 18 to 24 inches. Sawed-off lengths of pipe have open ends; pipes cut with a shear have pinched ends and are called pinch pipe. There are also manufactured marker pipes that are T-shaped rather than cylindrical in cross section. A commercial marker may consist of a copper-plated steel rod. All commercial markers have caps or heads that permit center punching for precise point location and stamping of the identifying information.

(e) Concrete Monument. Concrete monuments often have a short length of brass rod set in them to mark the exact location of the point. Federal surveying agencies using concrete monuments as permanent markers set identifying disks in them (see Figure 5-25).

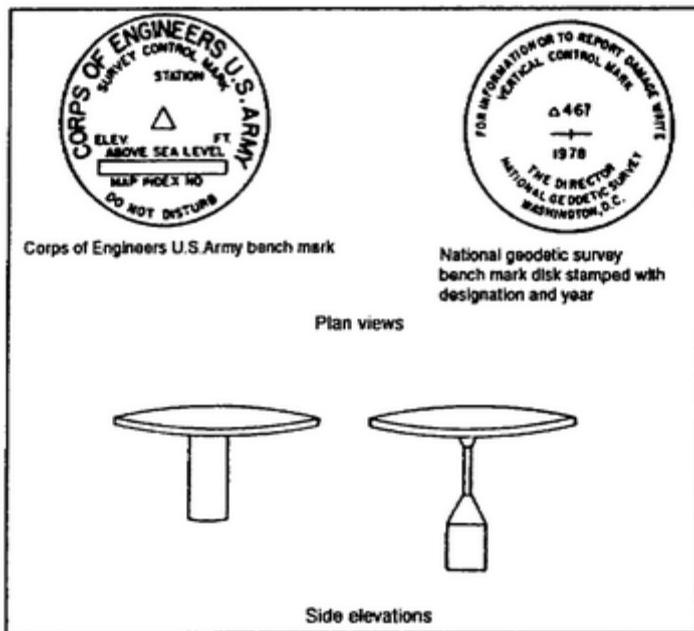


Figure 5-25. Brass Disks

(f) Brass Disk. Manufactured brass disks, similar to the ones shown in Figure 5-25, may be set in grouted holes in street pavements, sidewalks, steps, or the tops of retaining walls.

b. Marking Materials. Keel (lumber crayon) is a thick crayon used for marking stakes or other surfaces. Common marking devices that contain a quick-drying fluid and a felt tip are also popular for marking stakes. All of these types of graphic marking materials come in various colors.

In addition to keel, paint is used to mark pavement surfaces. Paint may be brushed on or sprayed from a spray can. To make the location of a point conspicuous, use a circle, a cross, or a triangle.

Identification symbols, such as station or traverse numbers, may also be painted on. For a neater job, stencils are sometimes used.

c. Flagging. Colored-cloth bunting or plastic tape is often used to make stakes conspicuous so that they will be easier to see. Flagging may also be used for identification purposes. For example, traverse stakes may be marked with one color, grade stakes with another. Red, yellow, orange, and white are the most popular flagging colors.

d. Note-Keeping Materials. Field notes are usually kept in a bound, standard field notebook. Sometimes loose-leaf notebooks are used but are not general recommended because of the chance of losing some pages. In the field notebook, the left-hand side of the page is used for recording measurement data and the right-hand side of the page is used for remark, sketches, and other supplementary information.

e. Personal Protective and Safety Equipment. In addition to the necessary field supplies and equipment, a field surveying party must carry all the necessary items of personal protective equipment such as containers for drinking water, first-aid kits, gloves, and wet-weather gear, as needed, since they usually work a considerable distance away from the main operational base. For example, if you happen to be taping through a marsh filled with icy water, you would not have a chance to return to the base to get your rubber boots.

In construction areas where the assigned personnel are required to wear hard hats, often, you are also required to wear a hard hat. Be prepared for any situation. Study the situation in advance, considering both the physical and environmental conditions.

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LESSON 5

PRACTICE EXERCISE

The following items will test your grasp of the material covered in this lesson. When you have completed the exercise, check your answer with the answer key that follows. If you answer any item incorrectly, study again that part which contains the portion involved.

1. What type of viewing magnification does the hand level have?
 - A. Two times
 - B. Reversed magnification
 - C. Short magnification
 - D. No magnification

2. How many leveling screws are on the automatic level?
 - A. Three
 - B. Four
 - C. Two
 - D. One

3. When should you make use of holes or cracks in the ground for the tripod legs when setting up the tripod?
 - A. When the tripod is set up on a hill.
 - B. When the tripod is set up on a smooth or slippery surface.
 - C. When the tripod is used in conjunction with a range pole.
 - D. Every time the tripod is set up.

4. When would it be recommended to use a target on a leveling rod?
 - A. When sights are obstructed through leaves and brush.
 - B. When a constant grade is needed from one instrument setup.
 - C. When there is poor visibility.
 - D. All of the above.

5. How is a 2- by 2-inch semipermanent wooden hub placed?
 - A. Flush or almost flush with the ground.
 - B. As a guard stake.
 - C. As the centerline of a road.
 - D. As a grade stake.

LESSON 5

PRACTICE EXERCISE

ANSWER KEY AND FEEDBACK

Item	Correct answer and feedback
1.	D No magnification The hand level ... (page 5-6, para 5-1c(3) (b))
2.	A Three The automatic level has ... (page 5-4, para 5-1c(2))
3.	B When the tripod is set up on a smooth or slippery surface On smooth or ... (page 5-12, para 5-4a(2)(a))
4.	D All of the above The leveling rod ... (page 5-16, para 5-4h(1))
5.	A Flush or almost flush with the ground The principal distinction ... (page 5-20, para 5-5a(2))

APPENDIX A

LIST OF COMMON ACRONYMS

θ	A Greek letter used to identify an angle.
$\sqrt{}$	radical sign
"	second
\angle	angle
'	minute
+	plus
-	minus
:	ratio
=	equal
$\overline{}$	vinculum
°	degree
∴	therefore
÷	divide
ACCP	Army Correspondence Course Program
AIPD	Army Institute for Professional Development
AISI	automated integrated survey instrument
AMEDD	Army medical department
APO	air post office
app	appendix
AR	Army regulation
ASCE	American Society of Civil Engineers
attn	attention

AV	autovon
AWR	answer weight reference
CDC	Career Development Course
cos	cosine
cot	cotangent
csc	cosecant
DC	District of Columbia
DETC	distance education and training council
DINFOS	Defense Information School
DOD	Department of Defense
EDM	electronic distance measurement
elev	elevation
EN	engineer
EW	east-west
FM	field manual
ft	feet
GD	ground distance
HI	height of instrument
ICE	interservice correspondence exchange
inc	incorporation
IPD	Institute for Professional Development
JFK	John Fitzgerald Kennedy
MD	map distance
mil	A unit of angular measurement equal to 1/6400 of 360°.

MOS	military occupational specialty
NAVEDTRA	Naval Education Training Aide
no	number
NS	north-south
PX	post exchange
RCOAC	Reserved Component Officer's Advanced Course
RF	representative factor
RS	response sheet
RYE	retirement year ending
see	secant
SGT	sergeant
sin	sine
SS	signal subcourse
SSN	social security number
tan	tangent
TM	technical manual
TRADOC	US Army Training and Doctrine Command
US	United States
VA	Virginia

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APPENDIX B

RECOMMENDED READING LIST

CDC 3E551A. *Engineering Journeyman, Volume 4, Plane Surveying.* Undated.
FM 5-233. *Construction Surveying.* 4 January 1985.
NAVEDTRA 10696. *Engineering Aid 3.* September 1991.
TM 5-232. *Elements of Surveying.* 1 June 1971.

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APPENDIX C
NATURAL TRIGONOMETRIC-FUNCTIONS TABLES

D Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.00000	1.00000	0.00000	#####
1	0.00029	1.00000	0.00029	3437.74667
2	0.00058	1.00000	0.00058	1718.87319
3	0.00087	1.00000	0.00087	1145.91530
4	0.00116	1.00000	0.00116	859.43630
5	0.00145	1.00000	0.00145	687.54887
6	0.00175	1.00000	0.00175	572.95721
7	0.00204	1.00000	0.00204	491.10600
8	0.00233	1.00000	0.00233	429.71757
9	0.00262	1.00000	0.00262	381.97099
10	0.00291	1.00000	0.00291	343.77371
11	0.00320	0.99999	0.00320	312.52137
12	0.00349	0.99999	0.00349	286.47773
13	0.00378	0.99999	0.00378	264.44080
14	0.00407	0.99999	0.00407	245.55198
15	0.00436	0.99999	0.00436	229.18166
16	0.00465	0.99999	0.00465	214.85762
17	0.00495	0.99999	0.00495	202.21875
18	0.00524	0.99999	0.00524	190.98419
19	0.00553	0.99998	0.00553	180.93220
20	0.00582	0.99998	0.00582	171.88540
21	0.00611	0.99998	0.00611	163.70019
22	0.00640	0.99998	0.00640	156.25908
23	0.00669	0.99998	0.00669	149.46502
24	0.00698	0.99998	0.00698	143.23712
25	0.00727	0.99997	0.00727	137.50745
26	0.00756	0.99997	0.00756	132.21851
27	0.00785	0.99997	0.00785	127.32134
28	0.00814	0.99997	0.00815	122.77396
29	0.00844	0.99996	0.00844	118.54018
30	0.00873	0.99996	0.00873	114.58865
31	0.00902	0.99996	0.00902	110.89205
32	0.00931	0.99996	0.00931	107.42648
33	0.00960	0.99995	0.00960	104.17094
34	0.00989	0.99995	0.00989	101.10690
35	0.01018	0.99995	0.01018	98.21794
36	0.01047	0.99995	0.01047	95.48948
37	0.01076	0.99994	0.01076	92.90849
38	0.01105	0.99994	0.01105	90.46334
39	0.01134	0.99994	0.01135	88.14357
40	0.01164	0.99993	0.01164	85.93979
41	0.01193	0.99993	0.01193	83.84351
42	0.01222	0.99993	0.01222	81.84704
43	0.01251	0.99992	0.01251	79.94343
44	0.01280	0.99992	0.01280	78.12634
45	0.01309	0.99991	0.01309	76.39001
46	0.01338	0.99991	0.01338	74.72917
47	0.01367	0.99991	0.01367	73.13899
48	0.01396	0.99990	0.01396	71.61507
49	0.01425	0.99990	0.01425	70.15335
50	0.01454	0.99989	0.01455	68.75009
51	0.01483	0.99989	0.01484	67.40185
52	0.01513	0.99989	0.01513	66.10547
53	0.01542	0.99988	0.01542	64.85801
54	0.01571	0.99988	0.01571	63.65674
55	0.01600	0.99987	0.01600	62.49915
56	0.01629	0.99987	0.01629	61.38291
57	0.01658	0.99986	0.01658	60.30582
58	0.01687	0.99986	0.01687	59.26587
59	0.01716	0.99985	0.01716	58.26117
60	0.01745	0.99985	0.01746	57.28996
	Cos	Sin	Cot	Minutes
				Degrees

1 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.01745	0.99985	0.01746	57.28996
1	0.01774	0.99984	0.01775	56.35059
2	0.01803	0.99984	0.01804	55.44152
3	0.01832	0.99983	0.01833	54.56130
4	0.01862	0.99983	0.01862	53.70659
5	0.01891	0.99982	0.01891	52.88211
6	0.01920	0.99982	0.01920	52.06087
7	0.01949	0.99981	0.01949	51.30316
8	0.01978	0.99980	0.01978	50.54851
9	0.02007	0.99980	0.02007	49.81573
10	0.02036	0.99979	0.02036	49.10388
11	0.02065	0.99979	0.02066	48.41206
12	0.02094	0.99978	0.02095	47.73950
13	0.02123	0.99977	0.02124	47.08534
14	0.02152	0.99977	0.02153	46.44886
15	0.02181	0.99976	0.02182	45.82935
16	0.02211	0.99976	0.02211	45.22614
17	0.02240	0.99975	0.02240	44.63860
18	0.02269	0.99974	0.02269	44.06611
19	0.02298	0.99974	0.02298	43.50812
20	0.02327	0.99973	0.02328	42.96408
21	0.02356	0.99972	0.02357	42.43346
22	0.02385	0.99972	0.02386	41.91579
23	0.02414	0.99971	0.02415	41.41059
24	0.02443	0.99970	0.02444	40.91741
25	0.02472	0.99969	0.02473	40.43584
26	0.02501	0.99969	0.02502	39.96546
27	0.02530	0.99968	0.02531	39.50589
28	0.02560	0.99967	0.02560	39.05677
29	0.02589	0.99966	0.02589	38.61774
30	0.02618	0.99966	0.02619	38.18846
31	0.02647	0.99965	0.02648	37.76881
32	0.02676	0.99964	0.02677	37.35789
33	0.02705	0.99963	0.02706	36.95600
34	0.02734	0.99963	0.02735	36.56266
35	0.02763	0.99962	0.02764	36.17760
36	0.02792	0.99961	0.02793	35.80055
37	0.02821	0.99960	0.02822	35.43128
38	0.02850	0.99959	0.02851	35.06955
39	0.02879	0.99959	0.02881	34.71512
40	0.02908	0.99958	0.02910	34.36777
41	0.02938	0.99957	0.02939	34.02730
42	0.02967	0.99956	0.02968	33.69351
43	0.02996	0.99955	0.02997	33.36619
44	0.03025	0.99954	0.03026	33.04517
45	0.03054	0.99953	0.03055	32.73026
46	0.03083	0.99952	0.03084	32.42129
47	0.03112	0.99952	0.03114	32.11810
48	0.03141	0.99951	0.03143	31.82052
49	0.03170	0.99950	0.03172	31.52839
50	0.03199	0.99949	0.03201	31.24158
51	0.03228	0.99948	0.03230	30.95993
52	0.03257	0.99947	0.03259	30.68331
53	0.03286	0.99946	0.03288	30.41158
54	0.03316	0.99945	0.03317	30.14462
55	0.03345	0.99944	0.03348	29.88230
56	0.03374	0.99943	0.03376	29.62450
57	0.03403	0.99942	0.03405	29.37111
58	0.03432	0.99941	0.03434	29.12200
59	0.03461	0.99940	0.03463	28.87709
60	0.03490	0.99939	0.03492	28.63625

Cos	Sin	Cot	Tan	Minutes
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	2	Degrees			
Minutes		Sin	Cos	Tan	Cot
0	0.03490	0.99939	0.03492	28.63625	60
1	0.03519	0.99938	0.03521	28.39940	59
2	0.03548	0.99937	0.03550	28.16642	58
3	0.03577	0.99936	0.03579	27.93723	57
4	0.03606	0.99935	0.03609	27.71174	56
5	0.03635	0.99934	0.03638	27.48985	55
6	0.03664	0.99933	0.03667	27.27149	54
7	0.03693	0.99932	0.03696	27.05656	53
8	0.03723	0.99931	0.03725	26.84498	52
9	0.03752	0.99930	0.03754	26.63669	51
10	0.03781	0.99929	0.03783	26.43160	50
11	0.03810	0.99927	0.03812	26.22964	49
12	0.03839	0.99926	0.03842	26.03074	48
13	0.03868	0.99925	0.03871	25.83482	47
14	0.03897	0.99924	0.03900	25.64183	46
15	0.03926	0.99923	0.03929	25.45170	45
16	0.03955	0.99922	0.03958	25.26436	44
17	0.03984	0.99921	0.03987	25.07976	43
18	0.04013	0.99919	0.04016	24.89783	42
19	0.04042	0.99918	0.04046	24.71851	41
20	0.04071	0.99917	0.04075	24.54176	40
21	0.04100	0.99916	0.04104	24.36751	39
22	0.04129	0.99915	0.04133	24.19571	38
23	0.04159	0.99913	0.04162	24.02632	37
24	0.04188	0.99912	0.04191	23.85928	36
25	0.04217	0.99911	0.04220	23.69454	35
26	0.04246	0.99910	0.04250	23.53205	34
27	0.04275	0.99909	0.04279	23.37178	33
28	0.04304	0.99907	0.04308	23.21367	32
29	0.04333	0.99906	0.04337	23.05768	31
30	0.04362	0.99905	0.04366	22.90377	30
31	0.04391	0.99904	0.04395	22.75189	29
32	0.04420	0.99902	0.04424	22.60201	28
33	0.04449	0.99901	0.04454	22.45410	27
34	0.04478	0.99900	0.04483	22.30810	26
35	0.04507	0.99898	0.04512	22.16398	25
36	0.04536	0.99897	0.04541	22.02171	24
37	0.04565	0.99896	0.04570	21.88125	23
38	0.04594	0.99894	0.04599	21.74257	22
39	0.04623	0.99893	0.04628	21.60563	21
40	0.04653	0.99892	0.04658	21.47040	20
41	0.04682	0.99890	0.04687	21.33685	19
42	0.04711	0.99889	0.04716	21.20495	18
43	0.04740	0.99888	0.04745	21.07466	17
44	0.04769	0.99886	0.04774	20.94597	16
45	0.04798	0.99885	0.04803	20.81883	15
46	0.04827	0.99883	0.04833	20.69322	14
47	0.04856	0.99882	0.04862	20.56911	13
48	0.04885	0.99881	0.04891	20.44649	12
49	0.04914	0.99879	0.04920	20.32531	11
50	0.04943	0.99878	0.04949	20.20555	10
51	0.04972	0.99876	0.04978	20.08720	9
52	0.05001	0.99875	0.05007	19.97022	8
53	0.05030	0.99873	0.05037	19.85459	7
54	0.05059	0.99872	0.05066	19.74029	6
55	0.05088	0.99870	0.05095	19.62730	5
56	0.05117	0.99869	0.05124	19.51558	4
57	0.05146	0.99867	0.05153	19.40513	3
58	0.05175	0.99866	0.05182	19.29592	2
59	0.05205	0.99864	0.05212	19.18793	1
60	0.05234	0.99863	0.05241	19.06114	0

Cos	Sin	Cot	Tan	Minutes
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67 Degrees

	Minutes	3 Degree				
		Sin	Cos	Tan	Cot	
	0	0.05234	0.99863	0.05241	19.06114	60
	1	0.05263	0.99861	0.05270	18.97552	59
	2	0.05292	0.99860	0.05299	18.87107	58
	3	0.05321	0.99858	0.05328	18.76775	57
	4	0.05350	0.99857	0.05357	18.66556	56
	5	0.05379	0.99855	0.05387	18.56447	55
	6	0.05408	0.99854	0.05416	18.46447	54
	7	0.05437	0.99852	0.05445	18.36554	53
	8	0.05466	0.99851	0.05474	18.26765	52
	9	0.05495	0.99849	0.05503	18.17081	51
	10	0.05524	0.99847	0.05533	18.07498	50
	11	0.05553	0.99846	0.05562	17.98015	49
	12	0.05582	0.99844	0.05591	17.88631	48
	13	0.05611	0.99842	0.05620	17.79344	47
	14	0.05640	0.99841	0.05649	17.70153	46
	15	0.05669	0.99839	0.05678	17.61056	45
	16	0.05698	0.99838	0.05708	17.52052	44
	17	0.05727	0.99836	0.05737	17.43139	43
	18	0.05756	0.99834	0.05766	17.34315	42
	19	0.05785	0.99833	0.05795	17.25581	41
	20	0.05814	0.99831	0.05824	17.16934	40
	21	0.05844	0.99829	0.05854	17.08372	39
	22	0.05873	0.99827	0.05883	16.99896	38
	23	0.05902	0.99826	0.05912	16.91503	37
	24	0.05931	0.99824	0.05941	16.83191	36
	25	0.05960	0.99822	0.05970	16.74961	35
	26	0.05989	0.99821	0.05999	16.66811	34
	27	0.06018	0.99819	0.06029	16.58740	33
	28	0.06047	0.99817	0.06058	16.50746	32
	29	0.06076	0.99815	0.06087	16.42828	31
	30	0.06105	0.99813	0.06116	16.34986	30
	31	0.06134	0.99812	0.06145	16.27217	29
	32	0.06163	0.99810	0.06175	16.19523	28
	33	0.06192	0.99808	0.06204	16.11900	27
	34	0.06221	0.99806	0.06233	16.04348	26
	35	0.06250	0.99804	0.06262	15.96867	25
	36	0.06279	0.99803	0.06291	15.89454	24
	37	0.06308	0.99801	0.06321	15.82110	23
	38	0.06337	0.99799	0.06350	15.74834	22
	39	0.06366	0.99797	0.06379	15.67623	21
	40	0.06395	0.99795	0.06408	15.60478	20
	41	0.06424	0.99793	0.06438	15.53398	19
	42	0.06453	0.99792	0.06467	15.46381	18
	43	0.06482	0.99790	0.06496	15.39428	17
	44	0.06511	0.99788	0.06525	15.32536	16
	45	0.06540	0.99786	0.06554	15.25705	15
	46	0.06569	0.99784	0.06584	15.18935	14
	47	0.06598	0.99782	0.06613	15.12224	13
	48	0.06627	0.99780	0.06642	15.05572	12
	49	0.06656	0.99778	0.06671	14.98978	11
	50	0.06685	0.99776	0.06700	14.92442	10
	51	0.06714	0.99774	0.06730	14.85962	9
	52	0.06743	0.99772	0.06759	14.79537	8
	53	0.06773	0.99770	0.06788	14.73168	7
	54	0.06802	0.99768	0.06817	14.66853	6
	55	0.06831	0.99766	0.06847	14.60592	5
	56	0.06860	0.99764	0.06876	14.54383	4
	57	0.06889	0.99762	0.06905	14.48227	3
	58	0.06918	0.99760	0.06934	14.42123	2
	59	0.06947	0.99758	0.06963	14.36070	1
	60	0.06976	0.99756	0.06993	14.30067	0

Cos Sin Cot Tan Minutes

86 Degrees

4 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.06976	0.99756	0.06993	14.30067
1	0.07005	0.99754	0.07022	14.24113
2	0.07034	0.99752	0.07051	14.18209
3	0.07063	0.99750	0.07080	14.12354
4	0.07092	0.99748	0.07110	14.06546
5	0.07121	0.99746	0.07139	14.00786
6	0.07150	0.99744	0.07168	13.95072
7	0.07179	0.99742	0.07197	13.89405
8	0.07208	0.99740	0.07227	13.83783
9	0.07237	0.99738	0.07256	13.78206
10	0.07266	0.99736	0.07285	13.72674
11	0.07295	0.99734	0.07314	13.67186
12	0.07324	0.99731	0.07344	13.61741
13	0.07353	0.99729	0.07373	13.56339
14	0.07382	0.99727	0.07402	13.50980
15	0.07411	0.99725	0.07431	13.45663
16	0.07440	0.99723	0.07461	13.40387
17	0.07469	0.99721	0.07490	13.35152
18	0.07498	0.99719	0.07519	13.29957
19	0.07527	0.99716	0.07548	13.24803
20	0.07556	0.99714	0.07578	13.19688
21	0.07585	0.99712	0.07607	13.14613
22	0.07614	0.99710	0.07636	13.09576
23	0.07643	0.99708	0.07665	13.04577
24	0.07672	0.99705	0.07695	12.99616
25	0.07701	0.99703	0.07724	12.94692
26	0.07730	0.99701	0.07753	12.89806
27	0.07759	0.99699	0.07782	12.84956
28	0.07788	0.99696	0.07812	12.80142
29	0.07817	0.99694	0.07841	12.75363
30	0.07846	0.99692	0.07870	12.70620
31	0.07875	0.99689	0.07899	12.65912
32	0.07904	0.99687	0.07929	12.61239
33	0.07933	0.99685	0.07958	12.56600
34	0.07962	0.99683	0.07987	12.51994
35	0.07991	0.99680	0.08017	12.47422
36	0.08020	0.99678	0.08046	12.42883
37	0.08049	0.99676	0.08075	12.38377
38	0.08078	0.99673	0.08104	12.33903
39	0.08107	0.99671	0.08134	12.29461
40	0.08136	0.99668	0.08163	12.25051
41	0.08165	0.99666	0.08192	12.20672
42	0.08194	0.99664	0.08221	12.16324
43	0.08223	0.99661	0.08251	12.12006
44	0.08252	0.99659	0.08280	12.07719
45	0.08281	0.99657	0.08309	12.03462
46	0.08310	0.99654	0.08339	11.99235
47	0.08339	0.99652	0.08368	11.95037
48	0.08368	0.99649	0.08397	11.90868
49	0.08397	0.99647	0.08427	11.86728
50	0.08426	0.99644	0.08456	11.82617
51	0.08455	0.99642	0.08485	11.78533
52	0.08484	0.99639	0.08514	11.74478
53	0.08513	0.99637	0.08544	11.70450
54	0.08542	0.99635	0.08573	11.66450
55	0.08571	0.99632	0.08602	11.62476
56	0.08600	0.99630	0.08632	11.58529
57	0.08629	0.99627	0.08661	11.54609
58	0.08658	0.99625	0.08690	11.50715
59	0.08687	0.99622	0.08720	11.46847
60	0.08716	0.99619	0.08749	11.43005

Cos	Sin	Cot	Tan	Minutes
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5 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.08716	0.99619	0.08749	11.43005
1	0.08745	0.99617	0.08778	11.39188
2	0.08774	0.99614	0.08807	11.35397
3	0.08803	0.99612	0.08837	11.31630
4	0.08831	0.99609	0.08866	11.27889
5	0.08860	0.99607	0.08895	11.24171
6	0.08889	0.99604	0.08925	11.20478
7	0.08918	0.99602	0.08954	11.16809
8	0.08947	0.99599	0.08983	11.13163
9	0.08976	0.99596	0.09013	11.09542
10	0.09005	0.99594	0.09042	11.05943
11	0.09034	0.99591	0.09071	11.02368
12	0.09063	0.99588	0.09101	10.98815
13	0.09092	0.99586	0.09130	10.95285
14	0.09121	0.99583	0.09159	10.91777
15	0.09150	0.99580	0.09189	10.88292
16	0.09179	0.99578	0.09218	10.84829
17	0.09208	0.99575	0.09247	10.81387
18	0.09237	0.99572	0.09277	10.77967
19	0.09266	0.99570	0.09306	10.74569
20	0.09295	0.99567	0.09335	10.71191
21	0.09324	0.99564	0.09365	10.67835
22	0.09353	0.99562	0.09394	10.64499
23	0.09382	0.99559	0.09423	10.61184
24	0.09411	0.99556	0.09453	10.57889
25	0.09440	0.99553	0.09482	10.54615
26	0.09469	0.99551	0.09511	10.51361
27	0.09498	0.99548	0.09541	10.48126
28	0.09527	0.99545	0.09570	10.44911
29	0.09556	0.99542	0.09600	10.41716
30	0.09585	0.99540	0.09629	10.38540
31	0.09614	0.99537	0.09658	10.35383
32	0.09642	0.99534	0.09688	10.32245
33	0.09671	0.99531	0.09717	10.29126
34	0.09700	0.99528	0.09746	10.26025
35	0.09729	0.99526	0.09776	10.22943
36	0.09758	0.99523	0.09805	10.19879
37	0.09787	0.99520	0.09834	10.16833
38	0.09816	0.99517	0.09864	10.13805
39	0.09845	0.99514	0.09893	10.10795
40	0.09874	0.99511	0.09923	10.07803
41	0.09903	0.99508	0.09952	10.04828
42	0.09932	0.99506	0.09981	10.01871
43	0.09961	0.99503	0.10011	9.98931
44	0.09990	0.99500	0.10040	9.96007
45	0.10019	0.99497	0.10069	9.93101
46	0.10048	0.99494	0.10099	9.90211
47	0.10077	0.99491	0.10128	9.87338
48	0.10106	0.99488	0.10158	9.84482
49	0.10135	0.99485	0.10187	9.81641
50	0.10164	0.99482	0.10216	9.78817
51	0.10192	0.99479	0.10246	9.76009
52	0.10221	0.99476	0.10275	9.73217
53	0.10250	0.99473	0.10305	9.70441
54	0.10279	0.99470	0.10334	9.67680
55	0.10308	0.99467	0.10363	9.64935
56	0.10337	0.99464	0.10393	9.62205
57	0.10366	0.99461	0.10422	9.59490
58	0.10395	0.99458	0.10452	9.56791
59	0.10424	0.99455	0.10481	9.54106
60	0.10453	0.99452	0.10510	9.51436

Cos	Sin	Cot	Tan	Minutes

84 Degrees

	6 Degree				
Minutes	Sin	Cos	Tan	Cot	Minutes
0	0.10453	0.99452	0.10510	9.51436	60
1	0.10482	0.99449	0.10540	9.48781	59
2	0.10511	0.99446	0.10569	9.46141	58
3	0.10540	0.99443	0.10599	9.43515	57
4	0.10569	0.99440	0.10628	9.40904	56
5	0.10597	0.99437	0.10657	9.38307	55
6	0.10626	0.99434	0.10687	9.35724	54
7	0.10655	0.99431	0.10716	9.33155	53
8	0.10684	0.99428	0.10746	9.30599	52
9	0.10713	0.99424	0.10775	9.28058	51
10	0.10742	0.99421	0.10805	9.25530	50
11	0.10771	0.99418	0.10834	9.23016	49
12	0.10800	0.99415	0.10863	9.20516	48
13	0.10829	0.99412	0.10893	9.18028	47
14	0.10858	0.99409	0.10922	9.15554	46
15	0.10887	0.99406	0.10952	9.13093	45
16	0.10916	0.99402	0.10981	9.10646	44
17	0.10945	0.99399	0.11011	9.08211	43
18	0.10973	0.99396	0.11040	9.05789	42
19	0.11002	0.99393	0.11070	9.03379	41
20	0.11031	0.99390	0.11099	9.00983	40
21	0.11060	0.99386	0.11128	8.98598	39
22	0.11089	0.99383	0.11158	8.96227	38
23	0.11118	0.99380	0.11187	8.93867	37
24	0.11147	0.99377	0.11217	8.91520	36
25	0.11176	0.99374	0.11246	8.89185	35
26	0.11205	0.99370	0.11276	8.86862	34
27	0.11234	0.99367	0.11305	8.84551	33
28	0.11263	0.99364	0.11335	8.82252	32
29	0.11291	0.99360	0.11364	8.79964	31
30	0.11320	0.99357	0.11394	8.77689	30
31	0.11349	0.99354	0.11423	8.75425	29
32	0.11378	0.99351	0.11452	8.73172	28
33	0.11407	0.99347	0.11482	8.70931	27
34	0.11436	0.99344	0.11511	8.68701	26
35	0.11465	0.99341	0.11541	8.66482	25
36	0.11494	0.99337	0.11570	8.64275	24
37	0.11523	0.99334	0.11600	8.62078	23
38	0.11552	0.99331	0.11629	8.59893	22
39	0.11580	0.99327	0.11659	8.57718	21
40	0.11609	0.99324	0.11688	8.55555	20
41	0.11638	0.99320	0.11718	8.53402	19
42	0.11667	0.99317	0.11747	8.51259	18
43	0.11696	0.99314	0.11777	8.49128	17
44	0.11725	0.99310	0.11806	8.47007	16
45	0.11754	0.99307	0.11836	8.44896	15
46	0.11783	0.99303	0.11865	8.42795	14
47	0.11812	0.99300	0.11895	8.40705	13
48	0.11840	0.99297	0.11924	8.38625	12
49	0.11869	0.99293	0.11954	8.36555	11
50	0.11898	0.99290	0.11983	8.34496	10
51	0.11927	0.99286	0.12013	8.32446	9
52	0.11956	0.99283	0.12042	8.30406	8
53	0.11985	0.99279	0.12072	8.28376	7
54	0.12014	0.99276	0.12101	8.26355	6
55	0.12043	0.99272	0.12131	8.24345	5
56	0.12071	0.99269	0.12160	8.22344	4
57	0.12100	0.99265	0.12190	8.20352	3
58	0.12129	0.99262	0.12219	8.18370	2
59	0.12158	0.99258	0.12249	8.16398	1
60	0.12187	0.99255	0.12278	8.14435	0

Cos

Sin

Cot

Tan

Minutes

7 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.12187	0.99255	0.12278	8.14435	60
1	0.12216	0.99251	0.12308	8.12481	59
2	0.12245	0.99248	0.12338	8.10536	58
3	0.12274	0.99244	0.12367	8.08600	57
4	0.12302	0.99240	0.12397	8.06674	56
5	0.12331	0.99237	0.12426	8.04756	55
6	0.12360	0.99233	0.12456	8.02848	54
7	0.12389	0.99230	0.12485	8.00948	53
8	0.12418	0.99226	0.12515	7.99058	52
9	0.12447	0.99222	0.12544	7.97176	51
10	0.12476	0.99219	0.12574	7.95302	50
11	0.12504	0.99215	0.12603	7.93438	49
12	0.12533	0.99211	0.12633	7.91582	48
13	0.12562	0.99208	0.12662	7.89734	47
14	0.12591	0.99204	0.12692	7.87895	46
15	0.12620	0.99200	0.12722	7.86064	45
16	0.12649	0.99197	0.12751	7.84242	44
17	0.12678	0.99193	0.12781	7.82428	43
18	0.12706	0.99189	0.12810	7.80622	42
19	0.12735	0.99186	0.12840	7.78825	41
20	0.12764	0.99182	0.12869	7.77035	40
21	0.12793	0.99178	0.12899	7.75254	39
22	0.12822	0.99175	0.12929	7.73480	38
23	0.12851	0.99171	0.12958	7.71715	37
24	0.12880	0.99167	0.12988	7.69957	36
25	0.12908	0.99163	0.13017	7.68208	35
26	0.12937	0.99160	0.13047	7.66466	34
27	0.12966	0.99156	0.13076	7.64732	33
28	0.12995	0.99152	0.13106	7.63005	32
29	0.13024	0.99148	0.13136	7.61287	31
30	0.13053	0.99144	0.13165	7.59575	30
31	0.13081	0.99141	0.13195	7.57872	29
32	0.13110	0.99137	0.13224	7.56176	28
33	0.13139	0.99133	0.13254	7.54487	27
34	0.13168	0.99129	0.13284	7.52806	26
35	0.13197	0.99125	0.13313	7.51132	25
36	0.13226	0.99122	0.13343	7.49465	24
37	0.13254	0.99118	0.13372	7.47806	23
38	0.13283	0.99114	0.13402	7.46154	22
39	0.13312	0.99110	0.13432	7.44509	21
40	0.13341	0.99106	0.13461	7.42871	20
41	0.13370	0.99102	0.13491	7.41240	19
42	0.13399	0.99098	0.13521	7.39618	18
43	0.13427	0.99094	0.13550	7.37999	17
44	0.13456	0.99091	0.13580	7.36389	16
45	0.13485	0.99087	0.13609	7.34786	15
46	0.13514	0.99083	0.13639	7.33190	14
47	0.13543	0.99079	0.13669	7.31600	13
48	0.13572	0.99075	0.13698	7.30018	12
49	0.13600	0.99071	0.13728	7.28442	11
50	0.13629	0.99067	0.13758	7.26873	10
51	0.13658	0.99063	0.13787	7.25310	9
52	0.13687	0.99059	0.13817	7.23754	8
53	0.13716	0.99055	0.13846	7.22204	7
54	0.13744	0.99051	0.13876	7.20661	6
55	0.13773	0.99047	0.13906	7.19125	5
56	0.13802	0.99043	0.13935	7.17594	4
57	0.13831	0.99039	0.13965	7.16071	3
58	0.13860	0.99035	0.13995	7.14553	2
59	0.13889	0.99031	0.14024	7.13042	1
60	0.13917	0.99027	0.14054	7.11537	0

Cos

Sin

Cot

Tan

Minutes

82 Degrees

8 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.13917	0.99027	0.14054	7.11537
1	0.13946	0.99023	0.14084	7.10038
2	0.13975	0.99019	0.14113	7.08546
3	0.14004	0.99015	0.14143	7.07059
4	0.14033	0.99011	0.14173	7.05579
5	0.14061	0.99006	0.14202	7.04105
6	0.14090	0.99002	0.14232	7.02637
7	0.14119	0.98998	0.14262	7.01174
8	0.14148	0.98994	0.14291	6.99718
9	0.14177	0.98990	0.14321	6.98268
10	0.14205	0.98986	0.14351	6.96823
11	0.14234	0.98982	0.14381	6.95385
12	0.14263	0.98978	0.14410	6.93952
13	0.14292	0.98973	0.14440	6.92525
14	0.14320	0.98969	0.14470	6.91104
15	0.14349	0.98965	0.14499	6.89688
16	0.14378	0.98961	0.14529	6.88278
17	0.14407	0.98957	0.14559	6.86874
18	0.14436	0.98953	0.14588	6.85475
19	0.14464	0.98948	0.14618	6.84082
20	0.14493	0.98944	0.14648	6.82694
21	0.14522	0.98940	0.14678	6.81312
22	0.14551	0.98936	0.14707	6.79936
23	0.14580	0.98931	0.14737	6.78564
24	0.14608	0.98927	0.14767	6.77199
25	0.14637	0.98923	0.14796	6.75838
26	0.14666	0.98919	0.14826	6.74483
27	0.14695	0.98914	0.14856	6.73133
28	0.14723	0.98910	0.14886	6.71789
29	0.14752	0.98906	0.14915	6.70450
30	0.14781	0.98902	0.14945	6.69116
31	0.14810	0.98897	0.14975	6.67787
32	0.14838	0.98893	0.15005	6.66463
33	0.14867	0.98889	0.15034	6.65144
34	0.14896	0.98884	0.15064	6.63831
35	0.14925	0.98880	0.15094	6.62523
36	0.14954	0.98876	0.15124	6.61219
37	0.14982	0.98871	0.15153	6.59921
38	0.15011	0.98867	0.15183	6.58627
39	0.15040	0.98863	0.15213	6.57339
40	0.15069	0.98858	0.15243	6.56055
41	0.15097	0.98854	0.15272	6.54777
42	0.15126	0.98849	0.15302	6.53503
43	0.15155	0.98845	0.15332	6.52234
44	0.15184	0.98841	0.15362	6.50970
45	0.15212	0.98836	0.15391	6.49710
46	0.15241	0.98832	0.15421	6.48456
47	0.15270	0.98827	0.15451	6.47206
48	0.15299	0.98823	0.15481	6.45961
49	0.15327	0.98818	0.15511	6.44720
50	0.15356	0.98814	0.15540	6.43484
51	0.15385	0.98809	0.15570	6.42253
52	0.15414	0.98805	0.15600	6.41026
53	0.15442	0.98800	0.15630	6.39804
54	0.15471	0.98796	0.15660	6.38587
55	0.15500	0.98791	0.15689	6.37374
56	0.15529	0.98787	0.15719	6.36165
57	0.15557	0.98782	0.15749	6.34961
58	0.15586	0.98778	0.15779	6.33761
59	0.15615	0.98773	0.15809	6.32566
60	0.15643	0.98769	0.15838	6.31375

Cos	Sin	Cot	Tan	Minutes
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9 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.15643	0.98769	0.15838	6.31375
1	0.15672	0.98764	0.15868	6.30189
2	0.15701	0.98760	0.15898	6.29007
3	0.15730	0.98755	0.15928	6.27829
4	0.15758	0.98751	0.15958	6.26655
5	0.15787	0.98746	0.15988	6.25486
6	0.15816	0.98741	0.16017	6.24321
7	0.15845	0.98737	0.16047	6.23160
8	0.15873	0.98732	0.16077	6.22003
9	0.15902	0.98728	0.16107	6.20851
10	0.15931	0.98723	0.16137	6.19703
11	0.15959	0.98718	0.16167	6.18559
12	0.15988	0.98714	0.16196	6.17419
13	0.16017	0.98709	0.16226	6.16283
14	0.16046	0.98704	0.16256	6.15151
15	0.16074	0.98700	0.16286	6.14023
16	0.16103	0.98695	0.16318	6.12899
17	0.16132	0.98690	0.16346	6.11779
18	0.16160	0.98686	0.16376	6.10664
19	0.16189	0.98681	0.16405	6.09552
20	0.16218	0.98676	0.16435	6.08444
21	0.16246	0.98671	0.16465	6.07340
22	0.16275	0.98667	0.16495	6.06240
23	0.16304	0.98662	0.16525	6.05143
24	0.16333	0.98657	0.16555	6.04051
25	0.16361	0.98652	0.16585	6.02962
26	0.16390	0.98648	0.16615	6.01878
27	0.16419	0.98643	0.16645	6.00797
28	0.16447	0.98638	0.16674	5.99720
29	0.16476	0.98633	0.16704	5.98648
30	0.16505	0.98629	0.16734	5.97576
31	0.16533	0.98624	0.16764	5.96510
32	0.16562	0.98619	0.16794	5.95448
33	0.16591	0.98614	0.16824	5.94390
34	0.16620	0.98609	0.16854	5.93335
35	0.16648	0.98604	0.16884	5.92283
36	0.16677	0.98600	0.16914	5.91236
37	0.16706	0.98595	0.16944	5.90191
38	0.16734	0.98590	0.16974	5.89151
39	0.16763	0.98585	0.17004	5.88114
40	0.16792	0.98580	0.17033	5.87080
41	0.16820	0.98575	0.17063	5.86051
42	0.16849	0.98570	0.17093	5.85024
43	0.16878	0.98565	0.17123	5.84001
44	0.16906	0.98561	0.17153	5.82982
45	0.16935	0.98556	0.17183	5.81968
46	0.16964	0.98551	0.17213	5.80953
47	0.16992	0.98546	0.17243	5.79944
48	0.17021	0.98541	0.17273	5.78938
49	0.17050	0.98536	0.17303	5.77936
50	0.17078	0.98531	0.17333	5.76937
51	0.17107	0.98526	0.17363	5.75941
52	0.17136	0.98521	0.17393	5.74949
53	0.17164	0.98516	0.17423	5.73960
54	0.17193	0.98511	0.17453	5.72974
55	0.17222	0.98506	0.17483	5.71992
56	0.17250	0.98501	0.17513	5.71013
57	0.17279	0.98496	0.17543	5.70037
58	0.17308	0.98491	0.17573	5.69064
59	0.17336	0.98486	0.17603	5.68094
60	0.17365	0.98481	0.17633	5.67128

Cos	Sin	Cot	Tan	Minutes
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80 Degrees

10 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.17365	0.98481	0.17633	5.67128
1	0.17393	0.98476	0.17663	5.66165
2	0.17422	0.98471	0.17693	5.65205
3	0.17451	0.98466	0.17723	5.64248
4	0.17479	0.98461	0.17753	5.63295
5	0.17508	0.98455	0.17783	5.62344
6	0.17537	0.98450	0.17813	5.61397
7	0.17565	0.98445	0.17843	5.60452
8	0.17594	0.98440	0.17873	5.59511
9	0.17623	0.98435	0.17903	5.58573
10	0.17651	0.98430	0.17933	5.57638
11	0.17680	0.98425	0.17963	5.56706
12	0.17708	0.98420	0.17993	5.55777
13	0.17737	0.98414	0.18023	5.54851
14	0.17766	0.98409	0.18053	5.53927
15	0.17794	0.98404	0.18083	5.53007
16	0.17823	0.98399	0.18113	5.52090
17	0.17852	0.98394	0.18143	5.51176
18	0.17880	0.98389	0.18173	5.50264
19	0.17909	0.98383	0.18203	5.49356
20	0.17937	0.98378	0.18233	5.48451
21	0.17966	0.98373	0.18263	5.47548
22	0.17995	0.98368	0.18293	5.46648
23	0.18023	0.98362	0.18323	5.45751
24	0.18052	0.98357	0.18353	5.44857
25	0.18081	0.98352	0.18384	5.43966
26	0.18109	0.98347	0.18414	5.43077
27	0.18138	0.98341	0.18444	5.42192
28	0.18166	0.98336	0.18474	5.41309
29	0.18195	0.98331	0.18504	5.40429
30	0.18224	0.98325	0.18534	5.39552
31	0.18252	0.98320	0.18564	5.38677
32	0.18281	0.98315	0.18594	5.37805
33	0.18309	0.98310	0.18624	5.36936
34	0.18338	0.98304	0.18654	5.36070
35	0.18367	0.98299	0.18684	5.35206
36	0.18395	0.98294	0.18714	5.34345
37	0.18424	0.98288	0.18745	5.33487
38	0.18452	0.98283	0.18775	5.32631
39	0.18481	0.98277	0.18805	5.31778
40	0.18509	0.98272	0.18835	5.30928
41	0.18538	0.98267	0.18865	5.30080
42	0.18567	0.98261	0.18895	5.29235
43	0.18595	0.98256	0.18925	5.28393
44	0.18624	0.98250	0.18955	5.27553
45	0.18652	0.98245	0.18986	5.26715
46	0.18681	0.98240	0.19016	5.25880
47	0.18710	0.98234	0.19046	5.25048
48	0.18738	0.98229	0.19076	5.24218
49	0.18767	0.98223	0.19106	5.23391
50	0.18795	0.98218	0.19136	5.22566
51	0.18824	0.98212	0.19166	5.21744
52	0.18852	0.98207	0.19197	5.20925
53	0.18881	0.98201	0.19227	5.20107
54	0.18910	0.98196	0.19257	5.19293
55	0.18938	0.98190	0.19287	5.18480
56	0.18967	0.98185	0.19317	5.17671
57	0.18995	0.98179	0.19347	5.16863
58	0.19024	0.98174	0.19378	5.16058
59	0.19052	0.98168	0.19408	5.15256
60	0.19081	0.98163	0.19438	5.14455

Cos	Sin	Cot	Tan	Minutes
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79 Degrees

11 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.19081	0.98163	0.19438	5.14455
1	0.19109	0.98157	0.19468	5.13658
2	0.19138	0.98152	0.19498	5.12882
3	0.19167	0.98146	0.19529	5.12069
4	0.19195	0.98140	0.19559	5.11279
5	0.19224	0.98135	0.19589	5.10490
6	0.19252	0.98129	0.19619	5.09704
7	0.19281	0.98124	0.19649	5.08921
8	0.19309	0.98118	0.19680	5.08139
9	0.19338	0.98112	0.19710	5.07360
10	0.19366	0.98107	0.19740	5.06584
11	0.19395	0.98101	0.19770	5.05809
12	0.19423	0.98096	0.19801	5.05037
13	0.19452	0.98090	0.19831	5.04267
14	0.19481	0.98084	0.19861	5.03499
15	0.19509	0.98079	0.19891	5.02734
16	0.19538	0.98073	0.19921	5.01971
17	0.19566	0.98067	0.19952	5.01210
18	0.19595	0.98061	0.19982	5.00451
19	0.19623	0.98056	0.20012	4.99695
20	0.19652	0.98050	0.20042	4.98940
21	0.19680	0.98044	0.20073	4.98188
22	0.19709	0.98039	0.20103	4.97438
23	0.19737	0.98033	0.20133	4.96690
24	0.19766	0.98027	0.20164	4.95945
25	0.19794	0.98021	0.20194	4.95201
26	0.19823	0.98016	0.20224	4.94460
27	0.19851	0.98010	0.20254	4.93721
28	0.19880	0.98004	0.20285	4.92984
29	0.19908	0.97998	0.20315	4.92249
30	0.19937	0.97992	0.20345	4.91516
31	0.19965	0.97987	0.20376	4.90785
32	0.19994	0.97981	0.20406	4.90056
33	0.20022	0.97975	0.20436	4.89330
34	0.20051	0.97969	0.20466	4.88605
35	0.20079	0.97963	0.20497	4.87882
36	0.20108	0.97958	0.20527	4.87162
37	0.20136	0.97952	0.20557	4.86444
38	0.20165	0.97946	0.20588	4.85727
39	0.20193	0.97940	0.20618	4.85013
40	0.20222	0.97934	0.20648	4.84300
41	0.20250	0.97928	0.20679	4.83590
42	0.20279	0.97922	0.20709	4.82882
43	0.20307	0.97916	0.20739	4.82175
44	0.20336	0.97910	0.20770	4.81471
45	0.20364	0.97905	0.20800	4.80769
46	0.20393	0.97899	0.20830	4.80068
47	0.20421	0.97893	0.20861	4.79370
48	0.20450	0.97887	0.20891	4.78673
49	0.20478	0.97881	0.20921	4.77978
50	0.20507	0.97875	0.20952	4.77286
51	0.20535	0.97869	0.20982	4.76595
52	0.20563	0.97863	0.21013	4.75906
53	0.20592	0.97857	0.21043	4.75219
54	0.20620	0.97851	0.21073	4.74534
55	0.20649	0.97845	0.21104	4.73851
56	0.20677	0.97839	0.21134	4.73170
57	0.20706	0.97833	0.21164	4.72490
58	0.20734	0.97827	0.21195	4.71813
59	0.20763	0.97821	0.21225	4.71137
60	0.20791	0.97815	0.21256	4.70463

Cos	Sin	Cot	Tan	Minutes
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12 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.20791	0.97815	0.21256	4.70463
1	0.20820	0.97809	0.21286	4.69791
2	0.20848	0.97803	0.21316	4.69121
3	0.20877	0.97797	0.21347	4.68452
4	0.20905	0.97791	0.21377	4.67786
5	0.20933	0.97784	0.21408	4.67121
6	0.20962	0.97778	0.21438	4.66458
7	0.20990	0.97772	0.21469	4.65797
8	0.21019	0.97766	0.21499	4.65138
9	0.21047	0.97760	0.21529	4.64480
10	0.21076	0.97754	0.21560	4.63825
11	0.21104	0.97748	0.21590	4.63171
12	0.21132	0.97742	0.21621	4.62518
13	0.21161	0.97735	0.21651	4.61868
14	0.21189	0.97729	0.21682	4.61219
15	0.21218	0.97723	0.21712	4.60572
16	0.21246	0.97717	0.21743	4.59927
17	0.21275	0.97711	0.21773	4.59283
18	0.21303	0.97705	0.21804	4.58641
19	0.21331	0.97698	0.21834	4.58001
20	0.21360	0.97692	0.21864	4.57363
21	0.21388	0.97686	0.21895	4.56726
22	0.21417	0.97680	0.21925	4.56091
23	0.21445	0.97673	0.21956	4.55458
24	0.21474	0.97667	0.21986	4.54826
25	0.21502	0.97661	0.22017	4.54196
26	0.21530	0.97655	0.22047	4.53568
27	0.21559	0.97648	0.22078	4.52941
28	0.21587	0.97642	0.22108	4.52316
29	0.21615	0.97636	0.22139	4.51693
30	0.21644	0.97630	0.22169	4.51071
31	0.21672	0.97623	0.22200	4.50451
32	0.21701	0.97617	0.22231	4.49832
33	0.21729	0.97611	0.22261	4.49215
34	0.21758	0.97604	0.22292	4.48600
35	0.21786	0.97598	0.22322	4.47986
36	0.21814	0.97592	0.22353	4.47374
37	0.21843	0.97585	0.22383	4.46764
38	0.21871	0.97579	0.22414	4.46155
39	0.21899	0.97573	0.22444	4.45548
40	0.21928	0.97566	0.22475	4.44942
41	0.21956	0.97560	0.22505	4.44338
42	0.21985	0.97553	0.22536	4.43735
43	0.22013	0.97547	0.22567	4.43134
44	0.22041	0.97541	0.22597	4.42534
45	0.22070	0.97534	0.22628	4.41936
46	0.22098	0.97528	0.22658	4.41340
47	0.22126	0.97521	0.22689	4.40745
48	0.22155	0.97515	0.22719	4.40152
49	0.22183	0.97508	0.22750	4.39560
50	0.22212	0.97502	0.22781	4.38969
51	0.22240	0.97496	0.22811	4.38381
52	0.22268	0.97489	0.22842	4.37793
53	0.22297	0.97483	0.22872	4.37207
54	0.22325	0.97476	0.22903	4.36623
55	0.22353	0.97470	0.22934	4.36040
56	0.22382	0.97463	0.22964	4.35459
57	0.22410	0.97457	0.22995	4.34879
58	0.22438	0.97450	0.23026	4.34300
59	0.22467	0.97444	0.23056	4.33723
60	0.22495	0.97437	0.23087	4.33148

Cos	Sin	Cot	Tan	Minutes
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77 Degrees

13 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.22495	0.97437	0.23087	4.33148
1	0.22523	0.97430	0.23117	4.32573
2	0.22552	0.97424	0.23148	4.32001
3	0.22580	0.97417	0.23179	4.31430
4	0.22608	0.97411	0.23209	4.30860
5	0.22637	0.97404	0.23240	4.30291
6	0.22665	0.97398	0.23271	4.29724
7	0.22693	0.97391	0.23301	4.29159
8	0.22722	0.97384	0.23332	4.28595
9	0.22750	0.97378	0.23363	4.28032
10	0.22778	0.97371	0.23393	4.27471
11	0.22807	0.97365	0.23424	4.26911
12	0.22835	0.97358	0.23455	4.26352
13	0.22863	0.97351	0.23485	4.25795
14	0.22892	0.97345	0.23516	4.25239
15	0.22920	0.97338	0.23547	4.24685
16	0.22948	0.97331	0.23578	4.24132
17	0.22977	0.97325	0.23608	4.23580
18	0.23005	0.97318	0.23639	4.23030
19	0.23033	0.97311	0.23670	4.22481
20	0.23062	0.97304	0.23700	4.21933
21	0.23090	0.97298	0.23731	4.21387
22	0.23118	0.97291	0.23762	4.20842
23	0.23146	0.97284	0.23793	4.20298
24	0.23175	0.97278	0.23823	4.19756
25	0.23203	0.97271	0.23854	4.19215
26	0.23231	0.97264	0.23885	4.18675
27	0.23260	0.97257	0.23916	4.18137
28	0.23288	0.97251	0.23946	4.17600
29	0.23316	0.97244	0.23977	4.17064
30	0.23345	0.97237	0.24008	4.16530
31	0.23373	0.97230	0.24039	4.15997
32	0.23401	0.97223	0.24069	4.15465
33	0.23429	0.97217	0.24100	4.14934
34	0.23458	0.97210	0.24131	4.14405
35	0.23486	0.97203	0.24162	4.13877
36	0.23514	0.97196	0.24193	4.13350
37	0.23542	0.97189	0.24223	4.12825
38	0.23571	0.97182	0.24254	4.12301
39	0.23599	0.97176	0.24285	4.11778
40	0.23627	0.97169	0.24316	4.11256
41	0.23656	0.97162	0.24347	4.10736
42	0.23684	0.97155	0.24377	4.10216
43	0.23712	0.97148	0.24408	4.09699
44	0.23740	0.97141	0.24439	4.09182
45	0.23769	0.97134	0.24470	4.08666
46	0.23797	0.97127	0.24501	4.08152
47	0.23825	0.97120	0.24532	4.07639
48	0.23853	0.97113	0.24562	4.07127
49	0.23882	0.97106	0.24593	4.06616
50	0.23910	0.97100	0.24624	4.06107
51	0.23938	0.97093	0.24655	4.05599
52	0.23966	0.97086	0.24686	4.05092
53	0.23995	0.97079	0.24717	4.04586
54	0.24023	0.97072	0.24747	4.04081
55	0.24051	0.97065	0.24778	4.03578
56	0.24079	0.97058	0.24809	4.03076
57	0.24108	0.97051	0.24840	4.02574
58	0.24136	0.97044	0.24871	4.02074
59	0.24164	0.97037	0.24902	4.01576
60	0.24192	0.97030	0.24933	4.01078

Cos	Sin	Cot	Tan	Minutes
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14 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.24192	0.97030	0.24933	4.01078	60
1	0.24220	0.97023	0.24964	4.00582	59
2	0.24249	0.97015	0.24995	4.00086	58
3	0.24277	0.97008	0.25026	3.99592	57
4	0.24305	0.97001	0.25056	3.99099	56
5	0.24333	0.96994	0.25087	3.98607	55
6	0.24362	0.96987	0.25118	3.98117	54
7	0.24390	0.96980	0.25149	3.97627	53
8	0.24418	0.96973	0.25180	3.97139	52
9	0.24446	0.96966	0.25211	3.96651	51
10	0.24474	0.96959	0.25242	3.96165	50
11	0.24503	0.96952	0.25273	3.95680	49
12	0.24531	0.96945	0.25304	3.95196	48
13	0.24559	0.96937	0.25335	3.94713	47
14	0.24587	0.96930	0.25366	3.94232	46
15	0.24615	0.96923	0.25397	3.93751	45
16	0.24644	0.96916	0.25428	3.93271	44
17	0.24672	0.96909	0.25459	3.92793	43
18	0.24700	0.96902	0.25490	3.92316	42
19	0.24728	0.96894	0.25521	3.91839	41
20	0.24756	0.96887	0.25552	3.91364	40
21	0.24784	0.96880	0.25583	3.90890	39
22	0.24813	0.96873	0.25614	3.90417	38
23	0.24841	0.96866	0.25645	3.89945	37
24	0.24869	0.96858	0.25676	3.89474	36
25	0.24897	0.96851	0.25707	3.89004	35
26	0.24925	0.96844	0.25738	3.88536	34
27	0.24954	0.96837	0.25769	3.88068	33
28	0.24982	0.96829	0.25800	3.87601	32
29	0.25010	0.96822	0.25831	3.87136	31
30	0.25038	0.96815	0.25862	3.86671	30
31	0.25066	0.96807	0.25893	3.86208	29
32	0.25094	0.96800	0.25924	3.85745	28
33	0.25122	0.96793	0.25955	3.85284	27
34	0.25151	0.96786	0.25986	3.84824	26
35	0.25179	0.96778	0.26017	3.84364	25
36	0.25207	0.96771	0.26048	3.83906	24
37	0.25235	0.96764	0.26079	3.83449	23
38	0.25263	0.96756	0.26110	3.82992	22
39	0.25291	0.96749	0.26141	3.82537	21
40	0.25320	0.96742	0.26172	3.82083	20
41	0.25348	0.96734	0.26203	3.81630	19
42	0.25376	0.96727	0.26235	3.81177	18
43	0.25404	0.96719	0.26266	3.80726	17
44	0.25432	0.96712	0.26297	3.80276	16
45	0.25460	0.96705	0.26328	3.79827	15
46	0.25488	0.96697	0.26359	3.79378	14
47	0.25516	0.96690	0.26390	3.78931	13
48	0.25545	0.96682	0.26421	3.78485	12
49	0.25573	0.96675	0.26452	3.78040	11
50	0.25601	0.96667	0.26483	3.77595	10
51	0.25629	0.96660	0.26515	3.77152	9
52	0.25657	0.96653	0.26546	3.76709	8
53	0.25685	0.96645	0.26577	3.76268	7
54	0.25713	0.96638	0.26608	3.75828	6
55	0.25741	0.96630	0.26639	3.75388	5
56	0.25769	0.96623	0.26670	3.74950	4
57	0.25798	0.96615	0.26701	3.74512	3
58	0.25826	0.96608	0.26733	3.74075	2
59	0.25854	0.96600	0.26764	3.73640	1
60	0.25882	0.96593	0.26795	3.73205	0

Cos	Sin	Cot	Tan	Minutes
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75 Degree

15 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.25882	0.96593	0.26795	3.73205
1	0.25910	0.96585	0.26826	3.72771
2	0.25938	0.96578	0.26857	3.72338
3	0.25966	0.96570	0.26888	3.71907
4	0.25994	0.96562	0.26920	3.71476
5	0.26022	0.96555	0.26951	3.71046
6	0.26050	0.96547	0.26982	3.70616
7	0.26079	0.96540	0.27013	3.70188
8	0.26107	0.96532	0.27044	3.69761
9	0.26135	0.96524	0.27076	3.69335
10	0.26163	0.96517	0.27107	3.68909
11	0.26191	0.96509	0.27138	3.68485
12	0.26219	0.96502	0.27169	3.68061
13	0.26247	0.96494	0.27201	3.67638
14	0.26275	0.96486	0.27232	3.67217
15	0.26303	0.96479	0.27263	3.66796
16	0.26331	0.96471	0.27294	3.66376
17	0.26359	0.96463	0.27326	3.65957
18	0.26387	0.96456	0.27357	3.65538
19	0.26415	0.96448	0.27388	3.65121
20	0.26443	0.96440	0.27419	3.64705
21	0.26471	0.96433	0.27451	3.64289
22	0.26500	0.96425	0.27482	3.63874
23	0.26528	0.96417	0.27513	3.63461
24	0.26556	0.96410	0.27545	3.63048
25	0.26584	0.96402	0.27576	3.62636
26	0.26612	0.96394	0.27607	3.62224
27	0.26640	0.96386	0.27638	3.61814
28	0.26668	0.96379	0.27670	3.61405
29	0.26696	0.96371	0.27701	3.60996
30	0.26724	0.96363	0.27732	3.60588
31	0.26752	0.96355	0.27764	3.60181
32	0.26780	0.96347	0.27795	3.59775
33	0.26808	0.96340	0.27826	3.59370
34	0.26836	0.96332	0.27858	3.58966
35	0.26864	0.96324	0.27889	3.58562
36	0.26892	0.96316	0.27921	3.58160
37	0.26920	0.96308	0.27952	3.57758
38	0.26948	0.96301	0.27983	3.57357
39	0.26976	0.96293	0.28015	3.56957
40	0.27004	0.96285	0.28046	3.56557
41	0.27032	0.96277	0.28077	3.56159
42	0.27060	0.96269	0.28109	3.55761
43	0.27088	0.96261	0.28140	3.55364
44	0.27116	0.96253	0.28172	3.54968
45	0.27144	0.96246	0.28203	3.54573
46	0.27172	0.96238	0.28234	3.54179
47	0.27200	0.96230	0.28266	3.53785
48	0.27228	0.96222	0.28297	3.53393
49	0.27256	0.96214	0.28329	3.53001
50	0.27284	0.96206	0.28360	3.52609
51	0.27312	0.96198	0.28391	3.52219
52	0.27340	0.96190	0.28423	3.51829
53	0.27368	0.96182	0.28454	3.51441
54	0.27396	0.96174	0.28486	3.51053
55	0.27424	0.96166	0.28517	3.50666
56	0.27452	0.96158	0.28549	3.50279
57	0.27480	0.96150	0.28580	3.49894
58	0.27508	0.96142	0.28612	3.49509
59	0.27536	0.96134	0.28643	3.49125
60	0.27564	0.96126	0.28675	3.48741

Cos	Sin	Cot	Tan	Minutes
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16 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.27564	0.96126	0.28675	3.48741	60
1	0.27592	0.96118	0.28706	3.48359	59
2	0.27620	0.96110	0.28738	3.47977	58
3	0.27648	0.96102	0.28769	3.47596	57
4	0.27676	0.96094	0.28801	3.47216	56
5	0.27704	0.96086	0.28832	3.46837	55
6	0.27731	0.96078	0.28864	3.46458	54
7	0.27759	0.96070	0.28895	3.46080	53
8	0.27787	0.96062	0.28927	3.45703	52
9	0.27815	0.96054	0.28958	3.45327	51
10	0.27843	0.96046	0.28990	3.44951	50
11	0.27871	0.96037	0.29021	3.44576	49
12	0.27899	0.96029	0.29053	3.44202	48
13	0.27927	0.96021	0.29084	3.43829	47
14	0.27955	0.96013	0.29116	3.43456	46
15	0.27983	0.96005	0.29147	3.43084	45
16	0.28011	0.95997	0.29179	3.42713	44
17	0.28039	0.95989	0.29210	3.42343	43
18	0.28067	0.95981	0.29242	3.41973	42
19	0.28095	0.95972	0.29274	3.41604	41
20	0.28123	0.95964	0.29305	3.41236	40
21	0.28150	0.95956	0.29337	3.40869	39
22	0.28178	0.95948	0.29368	3.40502	38
23	0.28206	0.95940	0.29400	3.40136	37
24	0.28234	0.95931	0.29432	3.39771	36
25	0.28262	0.95923	0.29463	3.39406	35
26	0.28290	0.95915	0.29495	3.39042	34
27	0.28318	0.95907	0.29526	3.38679	33
28	0.28346	0.95898	0.29558	3.38317	32
29	0.28374	0.95890	0.29590	3.37955	31
30	0.28402	0.95882	0.29621	3.37594	30
31	0.28429	0.95874	0.29653	3.37234	29
32	0.28457	0.95865	0.29685	3.36875	28
33	0.28485	0.95857	0.29716	3.36516	27
34	0.28513	0.95849	0.29748	3.36158	26
35	0.28541	0.95841	0.29780	3.35800	25
36	0.28569	0.95832	0.29811	3.35443	24
37	0.28597	0.95824	0.29843	3.35087	23
38	0.28625	0.95816	0.29875	3.34732	22
39	0.28652	0.95807	0.29906	3.34377	21
40	0.28680	0.95799	0.29938	3.34023	20
41	0.28708	0.95791	0.29970	3.33670	19
42	0.28736	0.95782	0.30001	3.33317	18
43	0.28764	0.95774	0.30033	3.32965	17
44	0.28792	0.95766	0.30065	3.32614	16
45	0.28820	0.95757	0.30097	3.32264	15
46	0.28847	0.95749	0.30128	3.31914	14
47	0.28875	0.95740	0.30160	3.31565	13
48	0.28903	0.95732	0.30192	3.31216	12
49	0.28931	0.95724	0.30224	3.30868	11
50	0.28959	0.95715	0.30255	3.30521	10
51	0.28987	0.95707	0.30287	3.30174	9
52	0.29015	0.95698	0.30319	3.29829	8
53	0.29042	0.95690	0.30351	3.29483	7
54	0.29070	0.95681	0.30382	3.29139	6
55	0.29098	0.95673	0.30414	3.28795	5
56	0.29126	0.95664	0.30446	3.28452	4
57	0.29154	0.95656	0.30478	3.28109	3
58	0.29182	0.95647	0.30509	3.27767	2
59	0.29209	0.95639	0.30541	3.27426	1
60	0.29237	0.95630	0.30573	3.27085	0

Cos

Sin

Cot

Tan

Minutes

17 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.29237	0.95630	0.30573	3.27085	60
1	0.29265	0.95622	0.30605	3.26745	59
2	0.29293	0.95613	0.30637	3.26406	58
3	0.29321	0.95605	0.30669	3.26067	57
4	0.29348	0.95596	0.30700	3.25729	56
5	0.29376	0.95588	0.30732	3.25392	55
6	0.29404	0.95579	0.30764	3.25055	54
7	0.29432	0.95571	0.30796	3.24719	53
8	0.29460	0.95562	0.30828	3.24383	52
9	0.29487	0.95554	0.30860	3.24049	51
10	0.29515	0.95545	0.30891	3.23714	50
11	0.29543	0.95536	0.30923	3.23381	49
12	0.29571	0.95528	0.30955	3.23048	48
13	0.29599	0.95519	0.30987	3.22715	47
14	0.29626	0.95511	0.31019	3.22384	46
15	0.29654	0.95502	0.31051	3.22053	45
16	0.29682	0.95493	0.31083	3.21722	44
17	0.29710	0.95485	0.31115	3.21392	43
18	0.29737	0.95476	0.31147	3.21063	42
19	0.29765	0.95467	0.31178	3.20734	41
20	0.29793	0.95459	0.31210	3.20406	40
21	0.29821	0.95450	0.31242	3.20079	39
22	0.29849	0.95441	0.31274	3.19752	38
23	0.29876	0.95433	0.31306	3.19426	37
24	0.29904	0.95424	0.31338	3.19100	36
25	0.29932	0.95415	0.31370	3.18775	35
26	0.29960	0.95407	0.31402	3.18451	34
27	0.29987	0.95398	0.31434	3.18127	33
28	0.30015	0.95389	0.31466	3.17804	32
29	0.30043	0.95380	0.31498	3.17481	31
30	0.30071	0.95372	0.31530	3.17159	30
31	0.30098	0.95363	0.31562	3.16838	29
32	0.30126	0.95354	0.31594	3.16517	28
33	0.30154	0.95345	0.31626	3.16197	27
34	0.30182	0.95337	0.31658	3.15877	26
35	0.30209	0.95328	0.31690	3.15558	25
36	0.30237	0.95319	0.31722	3.15240	24
37	0.30265	0.95310	0.31754	3.14922	23
38	0.30292	0.95301	0.31786	3.14605	22
39	0.30320	0.95293	0.31818	3.14288	21
40	0.30348	0.95284	0.31850	3.13972	20
41	0.30376	0.95275	0.31882	3.13656	19
42	0.30403	0.95266	0.31914	3.13341	18
43	0.30431	0.95257	0.31946	3.13027	17
44	0.30459	0.95248	0.31978	3.12713	16
45	0.30486	0.95240	0.32010	3.12400	15
46	0.30514	0.95231	0.32042	3.12087	14
47	0.30542	0.95222	0.32074	3.11775	13
48	0.30570	0.95213	0.32106	3.11464	12
49	0.30597	0.95204	0.32139	3.11153	11
50	0.30625	0.95195	0.32171	3.10842	10
51	0.30653	0.95186	0.32203	3.10532	9
52	0.30680	0.95177	0.32235	3.10223	8
53	0.30708	0.95168	0.32267	3.09914	7
54	0.30736	0.95159	0.32299	3.09606	6
55	0.30763	0.95150	0.32331	3.09298	5
56	0.30791	0.95142	0.32363	3.08991	4
57	0.30819	0.95133	0.32396	3.08685	3
58	0.30846	0.95124	0.32428	3.08379	2
59	0.30874	0.95115	0.32460	3.08073	1
60	0.30902	0.95106	0.32492	3.07768	0

Cos

Sin

Cot

Tan

Minutes

18 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.30902	0.95106	0.32492	3.07768
1	0.30929	0.95097	0.32524	3.07464
2	0.30957	0.95088	0.32556	3.07160
3	0.30985	0.95079	0.32588	3.06857
4	0.31012	0.95070	0.32621	3.06554
5	0.31040	0.95061	0.32653	3.06252
6	0.31068	0.95052	0.32685	3.05950
7	0.31095	0.95043	0.32717	3.05649
8	0.31123	0.95033	0.32749	3.05349
9	0.31151	0.95024	0.32782	3.05049
10	0.31178	0.95015	0.32814	3.04749
11	0.31206	0.95006	0.32846	3.04450
12	0.31233	0.94997	0.32878	3.04152
13	0.31261	0.94988	0.32911	3.03854
14	0.31289	0.94979	0.32943	3.03556
15	0.31316	0.94970	0.32975	3.03260
16	0.31344	0.94961	0.33007	3.02963
17	0.31372	0.94952	0.33040	3.02667
18	0.31399	0.94943	0.33072	3.02372
19	0.31427	0.94933	0.33104	3.02077
20	0.31454	0.94924	0.33136	3.01783
21	0.31482	0.94915	0.33169	3.01489
22	0.31510	0.94906	0.33201	3.01196
23	0.31537	0.94897	0.33233	3.00903
24	0.31565	0.94888	0.33266	3.00611
25	0.31593	0.94878	0.33298	3.00319
26	0.31620	0.94869	0.33330	3.00028
27	0.31648	0.94860	0.33363	2.99738
28	0.31675	0.94851	0.33395	2.99447
29	0.31703	0.94842	0.33427	2.99158
30	0.31730	0.94832	0.33460	2.98868
31	0.31758	0.94823	0.33492	2.98580
32	0.31786	0.94814	0.33524	2.98292
33	0.31813	0.94805	0.33557	2.98004
34	0.31841	0.94795	0.33589	2.97717
35	0.31868	0.94786	0.33621	2.97430
36	0.31896	0.94777	0.33654	2.97144
37	0.31923	0.94768	0.33686	2.96858
38	0.31951	0.94758	0.33718	2.96573
39	0.31979	0.94749	0.33751	2.96288
40	0.32006	0.94740	0.33783	2.96004
41	0.32034	0.94730	0.33816	2.95721
42	0.32061	0.94721	0.33848	2.95437
43	0.32089	0.94712	0.33881	2.95155
44	0.32116	0.94702	0.33913	2.94872
45	0.32144	0.94693	0.33945	2.94591
46	0.32171	0.94684	0.33978	2.94309
47	0.32199	0.94674	0.34010	2.94028
48	0.32227	0.94665	0.34043	2.93748
49	0.32254	0.94656	0.34075	2.93468
50	0.32282	0.94646	0.34108	2.93189
51	0.32309	0.94637	0.34140	2.92910
52	0.32337	0.94627	0.34173	2.92632
53	0.32364	0.94618	0.34205	2.92354
54	0.32392	0.94609	0.34238	2.92076
55	0.32419	0.94599	0.34270	2.91799
56	0.32447	0.94590	0.34303	2.91523
57	0.32474	0.94580	0.34335	2.91246
58	0.32502	0.94571	0.34368	2.90971
59	0.32529	0.94561	0.34400	2.90696
60	0.32557	0.94552	0.34433	2.90421

Cos	Sin	Cot	Tan	Minutes
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71 Degrees

19 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.32557	0.94552	0.34433	2.90421
1	0.32584	0.94542	0.34465	2.90147
2	0.32612	0.94533	0.34498	2.89873
3	0.32639	0.94523	0.34530	2.89600
4	0.32667	0.94514	0.34563	2.89327
5	0.32694	0.94504	0.34596	2.89055
6	0.32722	0.94495	0.34628	2.88783
7	0.32749	0.94485	0.34661	2.88511
8	0.32777	0.94476	0.34693	2.88240
9	0.32804	0.94466	0.34726	2.87970
10	0.32832	0.94457	0.34758	2.87700
11	0.32859	0.94447	0.34791	2.87430
12	0.32887	0.94438	0.34824	2.87161
13	0.32914	0.94428	0.34856	2.86892
14	0.32942	0.94418	0.34889	2.86624
15	0.32969	0.94409	0.34922	2.86356
16	0.32997	0.94399	0.34954	2.86089
17	0.33024	0.94390	0.34987	2.85822
18	0.33051	0.94380	0.35020	2.85555
19	0.33079	0.94370	0.35052	2.85289
20	0.33106	0.94361	0.35085	2.85023
21	0.33134	0.94351	0.35118	2.84758
22	0.33161	0.94342	0.35150	2.84494
23	0.33189	0.94332	0.35183	2.84229
24	0.33216	0.94322	0.35216	2.83965
25	0.33244	0.94313	0.35248	2.83702
26	0.33271	0.94303	0.35281	2.83439
27	0.33298	0.94293	0.35314	2.83176
28	0.33326	0.94284	0.35346	2.82914
29	0.33353	0.94274	0.35379	2.82653
30	0.33381	0.94264	0.35412	2.82391
31	0.33408	0.94254	0.35445	2.82130
32	0.33436	0.94245	0.35477	2.81870
33	0.33463	0.94235	0.35510	2.81610
34	0.33490	0.94225	0.35543	2.81350
35	0.33518	0.94215	0.35576	2.81091
36	0.33545	0.94206	0.35608	2.80833
37	0.33573	0.94196	0.35641	2.80574
38	0.33600	0.94186	0.35674	2.80316
39	0.33627	0.94176	0.35707	2.80059
40	0.33655	0.94167	0.35740	2.79802
41	0.33682	0.94157	0.35772	2.79545
42	0.33710	0.94147	0.35805	2.79289
43	0.33737	0.94137	0.35838	2.79033
44	0.33764	0.94127	0.35871	2.78778
45	0.33792	0.94118	0.35904	2.78523
46	0.33819	0.94108	0.35937	2.78269
47	0.33846	0.94098	0.35969	2.78014
48	0.33874	0.94088	0.36002	2.77761
49	0.33901	0.94078	0.36035	2.77507
50	0.33929	0.94068	0.36068	2.77254
51	0.33956	0.94058	0.36101	2.77002
52	0.33983	0.94049	0.36134	2.76750
53	0.34011	0.94039	0.36167	2.76498
54	0.34038	0.94029	0.36199	2.76247
55	0.34065	0.94019	0.36232	2.75996
56	0.34093	0.94009	0.36265	2.75746
57	0.34120	0.93999	0.36298	2.75496
58	0.34147	0.93989	0.36331	2.75246
59	0.34175	0.93979	0.36364	2.74997
60	0.34202	0.93969	0.36397	2.74748

Cos	Sin	Cot	Tan	Minutes
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70 Degrees

20 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.34202	0.93969	0.36397	2.74748
1	0.34229	0.93959	0.36430	2.74499
2	0.34257	0.93949	0.36463	2.74251
3	0.34284	0.93939	0.36496	2.74004
4	0.34311	0.93929	0.36529	2.73756
5	0.34339	0.93919	0.36562	2.73509
6	0.34366	0.93909	0.36595	2.73263
7	0.34393	0.93899	0.36628	2.73017
8	0.34421	0.93889	0.36661	2.72771
9	0.34448	0.93879	0.36694	2.72526
10	0.34475	0.93869	0.36727	2.72281
11	0.34503	0.93859	0.36760	2.72036
12	0.34530	0.93849	0.36793	2.71792
13	0.34557	0.93839	0.36826	2.71548
14	0.34584	0.93829	0.36859	2.71305
15	0.34612	0.93819	0.36892	2.71062
16	0.34639	0.93809	0.36925	2.70819
17	0.34666	0.93799	0.36958	2.70577
18	0.34694	0.93789	0.36991	2.70335
19	0.34721	0.93779	0.37024	2.70094
20	0.34748	0.93769	0.37057	2.69853
21	0.34775	0.93759	0.37090	2.69612
22	0.34803	0.93748	0.37123	2.69371
23	0.34830	0.93738	0.37157	2.69131
24	0.34857	0.93728	0.37190	2.68892
25	0.34884	0.93718	0.37223	2.68653
26	0.34912	0.93708	0.37256	2.68414
27	0.34939	0.93698	0.37289	2.68175
28	0.34966	0.93688	0.37322	2.67937
29	0.34993	0.93677	0.37355	2.67700
30	0.35021	0.93667	0.37388	2.67462
31	0.35048	0.93657	0.37422	2.67225
32	0.35075	0.93647	0.37455	2.66989
33	0.35102	0.93637	0.37488	2.66752
34	0.35130	0.93626	0.37521	2.66516
35	0.35157	0.93616	0.37554	2.66281
36	0.35184	0.93606	0.37588	2.66046
37	0.35211	0.93596	0.37621	2.65811
38	0.35239	0.93585	0.37654	2.65576
39	0.35266	0.93575	0.37687	2.65342
40	0.35293	0.93565	0.37720	2.65109
41	0.35320	0.93555	0.37754	2.64875
42	0.35347	0.93544	0.37787	2.64642
43	0.35375	0.93534	0.37820	2.64410
44	0.35402	0.93524	0.37853	2.64177
45	0.35429	0.93514	0.37887	2.63945
46	0.35456	0.93503	0.37920	2.63714
47	0.35484	0.93493	0.37953	2.63483
48	0.35511	0.93483	0.37986	2.63252
49	0.35538	0.93472	0.38020	2.63021
50	0.35565	0.93462	0.38053	2.62791
51	0.35592	0.93452	0.38086	2.62561
52	0.35619	0.93441	0.38120	2.62332
53	0.35647	0.93431	0.38153	2.62103
54	0.35674	0.93420	0.38186	2.61874
55	0.35701	0.93410	0.38220	2.61646
56	0.35728	0.93400	0.38253	2.61418
57	0.35755	0.93389	0.38286	2.61190
58	0.35782	0.93379	0.38320	2.60963
59	0.35810	0.93368	0.38353	2.60736
60	0.35837	0.93358	0.38386	2.60509

Cos	Sin	Cot	Tan	Minutes

	21	Degrees			
Minutes	Sin	Cos	Tan	Cot	Minutes
0	0.35837	0.93358	0.38386	2.60509	60
1	0.35864	0.93348	0.38420	2.60283	59
2	0.35891	0.93337	0.38453	2.60057	58
3	0.35918	0.93327	0.38487	2.59831	57
4	0.35945	0.93316	0.38520	2.59606	56
5	0.35973	0.93306	0.38553	2.59381	55
6	0.36000	0.93295	0.38587	2.59156	54
7	0.36027	0.93285	0.38620	2.58932	53
8	0.36054	0.93274	0.38654	2.58708	52
9	0.36081	0.93264	0.38687	2.58484	51
10	0.36108	0.93253	0.38721	2.58261	50
11	0.36135	0.93243	0.38754	2.58038	49
12	0.36162	0.93232	0.38787	2.57815	48
13	0.36190	0.93222	0.38821	2.57593	47
14	0.36217	0.93211	0.38854	2.57371	46
15	0.36244	0.93201	0.38888	2.57150	45
16	0.36271	0.93190	0.38921	2.56928	44
17	0.36298	0.93180	0.38955	2.56707	43
18	0.36325	0.93169	0.38988	2.56487	42
19	0.36352	0.93159	0.39022	2.56266	41
20	0.36379	0.93148	0.39055	2.56046	40
21	0.36406	0.93137	0.39089	2.55827	39
22	0.36434	0.93127	0.39122	2.55608	38
23	0.36461	0.93116	0.39156	2.55389	37
24	0.36488	0.93106	0.39190	2.55170	36
25	0.36515	0.93095	0.39223	2.54952	35
26	0.36542	0.93084	0.39257	2.54734	34
27	0.36569	0.93074	0.39290	2.54516	33
28	0.36596	0.93063	0.39324	2.54299	32
29	0.36623	0.93052	0.39357	2.54082	31
30	0.36650	0.93042	0.39391	2.53865	30
31	0.36677	0.93031	0.39425	2.53648	29
32	0.36704	0.93020	0.39458	2.53432	28
33	0.36731	0.93010	0.39492	2.53217	27
34	0.36758	0.92999	0.39526	2.53001	26
35	0.36785	0.92988	0.39559	2.52786	25
36	0.36812	0.92978	0.39593	2.52571	24
37	0.36839	0.92967	0.39626	2.52357	23
38	0.36867	0.92956	0.39660	2.52142	22
39	0.36894	0.92945	0.39694	2.51929	21
40	0.36921	0.92935	0.39727	2.51715	20
41	0.36948	0.92924	0.39761	2.51502	19
42	0.36975	0.92913	0.39795	2.51289	18
43	0.37002	0.92902	0.39829	2.51076	17
44	0.37029	0.92892	0.39862	2.50864	16
45	0.37056	0.92881	0.39896	2.50652	15
46	0.37083	0.92870	0.39930	2.50440	14
47	0.37110	0.92859	0.39963	2.50229	13
48	0.37137	0.92849	0.39997	2.50018	12
49	0.37164	0.92838	0.40031	2.49807	11
50	0.37191	0.92827	0.40065	2.49597	10
51	0.37218	0.92816	0.40098	2.49386	9
52	0.37245	0.92805	0.40132	2.49177	8
53	0.37272	0.92794	0.40166	2.48967	7
54	0.37299	0.92784	0.40200	2.48758	6
55	0.37326	0.92773	0.40234	2.48549	5
56	0.37353	0.92762	0.40267	2.48340	4
57	0.37380	0.92751	0.40301	2.48132	3
58	0.37407	0.92740	0.40335	2.47924	2
59	0.37434	0.92729	0.40369	2.47716	1
60	0.37461	0.92718	0.40403	2.47509	0

Cos

Sin

Cot

Tan

Minutes

68 Degrees

	22	Degrees				
Minutes		Sin	Cos	Tan	Cot	Minutes
0	0.37461	0.92718	0.40403	2.47509		60
1	0.37488	0.92707	0.40436	2.47302		59
2	0.37515	0.92697	0.40470	2.47095		58
3	0.37542	0.92686	0.40504	2.46888		57
4	0.37569	0.92675	0.40538	2.46682		56
5	0.37595	0.92664	0.40572	2.46476		55
6	0.37622	0.92653	0.40606	2.46270		54
7	0.37649	0.92642	0.40640	2.46065		53
8	0.37676	0.92631	0.40674	2.45860		52
9	0.37703	0.92620	0.40707	2.45655		51
10	0.37730	0.92609	0.40741	2.45451		50
11	0.37757	0.92598	0.40775	2.45246		49
12	0.37784	0.92587	0.40809	2.45043		48
13	0.37811	0.92576	0.40843	2.44839		47
14	0.37838	0.92565	0.40877	2.44636		46
15	0.37865	0.92554	0.40911	2.44433		45
16	0.37892	0.92543	0.40945	2.44230		44
17	0.37919	0.92532	0.40979	2.44027		43
18	0.37946	0.92521	0.41013	2.43825		42
19	0.37973	0.92510	0.41047	2.43623		41
20	0.37999	0.92499	0.41081	2.43422		40
21	0.38026	0.92488	0.41115	2.43220		39
22	0.38053	0.92477	0.41149	2.43019		38
23	0.38080	0.92466	0.41183	2.42819		37
24	0.38107	0.92455	0.41217	2.42618		36
25	0.38134	0.92444	0.41251	2.42418		35
26	0.38161	0.92432	0.41285	2.42218		34
27	0.38188	0.92421	0.41319	2.42019		33
28	0.38215	0.92410	0.41353	2.41819		32
29	0.38241	0.92399	0.41387	2.41620		31
30	0.38268	0.92388	0.41421	2.41421		30
31	0.38295	0.92377	0.41455	2.41223		29
32	0.38322	0.92366	0.41490	2.41025		28
33	0.38349	0.92355	0.41524	2.40827		27
34	0.38376	0.92343	0.41558	2.40629		26
35	0.38403	0.92332	0.41592	2.40432		25
36	0.38430	0.92321	0.41626	2.40235		24
37	0.38456	0.92310	0.41660	2.40038		23
38	0.38483	0.92299	0.41694	2.39841		22
39	0.38510	0.92287	0.41728	2.39645		21
40	0.38537	0.92276	0.41763	2.39449		20
41	0.38564	0.92265	0.41797	2.39253		19
42	0.38591	0.92254	0.41831	2.39058		18
43	0.38617	0.92243	0.41865	2.38863		17
44	0.38644	0.92231	0.41899	2.38668		16
45	0.38671	0.92220	0.41933	2.38473		15
46	0.38698	0.92209	0.41968	2.38279		14
47	0.38725	0.92198	0.42002	2.38084		13
48	0.38752	0.92186	0.42036	2.37891		12
49	0.38778	0.92175	0.42070	2.37697		11
50	0.38805	0.92164	0.42105	2.37504		10
51	0.38832	0.92152	0.42139	2.37311		9
52	0.38859	0.92141	0.42173	2.37118		8
53	0.38886	0.92130	0.42207	2.36925		7
54	0.38912	0.92119	0.42242	2.36733		6
55	0.38939	0.92107	0.42276	2.36541		5
56	0.38966	0.92096	0.42310	2.36349		4
57	0.38993	0.92085	0.42345	2.36158		3
58	0.39020	0.92073	0.42379	2.35967		2
59	0.39046	0.92062	0.42413	2.35778		1
60	0.39073	0.92050	0.42447	2.35585		0

Cos Sin Cot Tan

Minutes

67 Degrees

23 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.39073	0.92050	0.42447	2.35585	60
1	0.39100	0.92039	0.42482	2.35395	59
2	0.39127	0.92028	0.42516	2.35205	58
3	0.39153	0.92016	0.42551	2.35015	57
4	0.39180	0.92005	0.42585	2.34825	56
5	0.39207	0.91994	0.42619	2.34636	55
6	0.39234	0.91982	0.42654	2.34447	54
7	0.39260	0.91971	0.42688	2.34258	53
8	0.39287	0.91959	0.42722	2.34069	52
9	0.39314	0.91948	0.42757	2.33881	51
10	0.39341	0.91936	0.42791	2.33693	50
11	0.39367	0.91925	0.42826	2.33505	49
12	0.39394	0.91914	0.42860	2.33317	48
13	0.39421	0.91902	0.42894	2.33130	47
14	0.39448	0.91891	0.42929	2.32943	46
15	0.39474	0.91879	0.42963	2.32756	45
16	0.39501	0.91868	0.42998	2.32570	44
17	0.39528	0.91856	0.43032	2.32383	43
18	0.39555	0.91845	0.43067	2.32197	42
19	0.39581	0.91833	0.43101	2.32012	41
20	0.39608	0.91822	0.43136	2.31826	40
21	0.39635	0.91810	0.43170	2.31641	39
22	0.39661	0.91799	0.43205	2.31456	38
23	0.39688	0.91787	0.43239	2.31271	37
24	0.39715	0.91775	0.43274	2.31086	36
25	0.39741	0.91764	0.43308	2.30902	35
26	0.39768	0.91752	0.43343	2.30718	34
27	0.39795	0.91741	0.43378	2.30534	33
28	0.39822	0.91729	0.43412	2.30351	32
29	0.39848	0.91718	0.43447	2.30167	31
30	0.39875	0.91706	0.43481	2.29984	30
31	0.39902	0.91694	0.43516	2.29801	29
32	0.39928	0.91683	0.43550	2.29619	28
33	0.39955	0.91671	0.43585	2.29437	27
34	0.39982	0.91660	0.43620	2.29254	26
35	0.40008	0.91648	0.43654	2.29073	25
36	0.40035	0.91636	0.43689	2.28891	24
37	0.40062	0.91625	0.43724	2.28710	23
38	0.40088	0.91613	0.43758	2.28528	22
39	0.40115	0.91601	0.43793	2.28348	21
40	0.40141	0.91590	0.43828	2.28167	20
41	0.40168	0.91578	0.43862	2.27987	19
42	0.40195	0.91566	0.43897	2.27806	18
43	0.40221	0.91555	0.43932	2.27626	17
44	0.40248	0.91543	0.43966	2.27447	16
45	0.40275	0.91531	0.44001	2.27267	15
46	0.40301	0.91519	0.44036	2.27088	14
47	0.40328	0.91508	0.44071	2.26909	13
48	0.40355	0.91496	0.44105	2.26730	12
49	0.40381	0.91484	0.44140	2.26552	11
50	0.40408	0.91472	0.44175	2.26374	10
51	0.40434	0.91461	0.44210	2.26196	9
52	0.40461	0.91449	0.44244	2.26018	8
53	0.40488	0.91437	0.44279	2.25840	7
54	0.40514	0.91425	0.44314	2.25663	6
55	0.40541	0.91414	0.44349	2.25486	5
56	0.40567	0.91402	0.44384	2.25309	4
57	0.40594	0.91390	0.44418	2.25132	3
58	0.40621	0.91378	0.44453	2.24956	2
59	0.40647	0.91366	0.44488	2.24780	1
60	0.40674	0.91355	0.44523	2.24604	0

Cos

Sin

Cot

Tan

Minutes

66

Degrees

24 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.40674	0.91355	0.44523	2.24604
1	0.40700	0.91343	0.44558	2.24428
2	0.40727	0.91331	0.44593	2.24252
3	0.40753	0.91319	0.44627	2.24077
4	0.40780	0.91307	0.44662	2.23902
5	0.40806	0.91295	0.44697	2.23727
6	0.40833	0.91283	0.44732	2.23553
7	0.40860	0.91272	0.44767	2.23378
8	0.40886	0.91260	0.44802	2.23204
9	0.40913	0.91248	0.44837	2.23030
10	0.40939	0.91236	0.44872	2.22857
11	0.40966	0.91224	0.44907	2.22683
12	0.40992	0.91212	0.44942	2.22510
13	0.41019	0.91200	0.44977	2.22337
14	0.41045	0.91188	0.45012	2.22164
15	0.41072	0.91176	0.45047	2.21992
16	0.41098	0.91164	0.45082	2.21819
17	0.41125	0.91152	0.45117	2.21647
18	0.41151	0.91140	0.45152	2.21475
19	0.41178	0.91128	0.45187	2.21304
20	0.41204	0.91116	0.45222	2.21132
21	0.41231	0.91104	0.45257	2.20961
22	0.41257	0.91092	0.45292	2.20790
23	0.41284	0.91080	0.45327	2.20619
24	0.41310	0.91068	0.45362	2.20449
25	0.41337	0.91056	0.45397	2.20278
26	0.41363	0.91044	0.45432	2.20108
27	0.41390	0.91032	0.45467	2.19938
28	0.41416	0.91020	0.45502	2.19769
29	0.41443	0.91008	0.45538	2.19599
30	0.41469	0.90996	0.45573	2.19430
31	0.41496	0.90984	0.45608	2.19261
32	0.41522	0.90972	0.45643	2.19092
33	0.41549	0.90960	0.45678	2.18923
34	0.41575	0.90948	0.45713	2.18755
35	0.41602	0.90936	0.45748	2.18587
36	0.41628	0.90924	0.45784	2.18419
37	0.41655	0.90911	0.45819	2.18251
38	0.41681	0.90899	0.45854	2.18084
39	0.41707	0.90887	0.45889	2.17916
40	0.41734	0.90875	0.45924	2.17749
41	0.41760	0.90863	0.45960	2.17582
42	0.41787	0.90851	0.45995	2.17416
43	0.41813	0.90839	0.46030	2.17249
44	0.41840	0.90826	0.46065	2.17083
45	0.41866	0.90814	0.46101	2.16917
46	0.41892	0.90802	0.46136	2.16751
47	0.41919	0.90790	0.46171	2.16585
48	0.41945	0.90778	0.46206	2.16420
49	0.41972	0.90766	0.46242	2.16255
50	0.41998	0.90753	0.46277	2.16090
51	0.42024	0.90741	0.46312	2.15925
52	0.42051	0.90729	0.46348	2.15760
53	0.42077	0.90717	0.46383	2.15596
54	0.42104	0.90704	0.46418	2.15432
55	0.42130	0.90692	0.46454	2.15268
56	0.42156	0.90680	0.46489	2.15104
57	0.42183	0.90668	0.46525	2.14940
58	0.42209	0.90655	0.46560	2.14777
59	0.42235	0.90643	0.46595	2.14614
60	0.42262	0.90631	0.46631	2.14451

Cos	Sin	Cot	Tan	Minutes
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65 Degrees

25 Degrees				
Minutes	Sin	Cos	Tan	Cot
0	0.42262	0.90631	0.46631	2.14451
1	0.42288	0.90618	0.46666	2.14268
2	0.42315	0.90606	0.46702	2.14125
3	0.42341	0.90594	0.46737	2.13963
4	0.42367	0.90582	0.46772	2.13801
5	0.42394	0.90569	0.46808	2.13639
6	0.42420	0.90557	0.46843	2.13477
7	0.42446	0.90545	0.46879	2.13316
8	0.42473	0.90532	0.46914	2.13154
9	0.42499	0.90520	0.46950	2.12993
10	0.42525	0.90507	0.46985	2.12832
11	0.42552	0.90495	0.47021	2.12671
12	0.42578	0.90483	0.47056	2.12511
13	0.42604	0.90470	0.47092	2.12350
14	0.42631	0.90458	0.47128	2.12190
15	0.42657	0.90446	0.47163	2.12030
16	0.42683	0.90433	0.47199	2.11871
17	0.42709	0.90421	0.47234	2.11711
18	0.42736	0.90408	0.47270	2.11552
19	0.42762	0.90396	0.47305	2.11392
20	0.42788	0.90383	0.47341	2.11233
21	0.42815	0.90371	0.47377	2.11075
22	0.42841	0.90358	0.47412	2.10916
23	0.42867	0.90346	0.47448	2.10758
24	0.42894	0.90334	0.47483	2.10600
25	0.42920	0.90321	0.47519	2.10442
26	0.42946	0.90309	0.47555	2.10284
27	0.42972	0.90296	0.47590	2.10126
28	0.42999	0.90284	0.47626	2.09969
29	0.43025	0.90271	0.47662	2.09811
30	0.43051	0.90259	0.47698	2.09654
31	0.43077	0.90246	0.47733	2.09498
32	0.43104	0.90233	0.47769	2.09341
33	0.43130	0.90221	0.47805	2.09184
34	0.43156	0.90208	0.47840	2.09028
35	0.43182	0.90196	0.47876	2.08872
36	0.43209	0.90183	0.47912	2.08716
37	0.43235	0.90171	0.47948	2.08560
38	0.43261	0.90158	0.47984	2.08405
39	0.43287	0.90146	0.48019	2.08250
40	0.43313	0.90133	0.48055	2.08094
41	0.43340	0.90120	0.48091	2.07939
42	0.43366	0.90108	0.48127	2.07785
43	0.43392	0.90095	0.48163	2.07630
44	0.43418	0.90082	0.48198	2.07476
45	0.43445	0.90070	0.48234	2.07321
46	0.43471	0.90057	0.48270	2.07167
47	0.43497	0.90045	0.48306	2.07014
48	0.43523	0.90032	0.48342	2.06860
49	0.43549	0.90019	0.48378	2.06706
50	0.43575	0.90007	0.48414	2.06553
51	0.43602	0.89994	0.48450	2.06400
52	0.43628	0.89981	0.48486	2.06247
53	0.43654	0.89968	0.48521	2.06094
54	0.43680	0.89956	0.48557	2.05942
55	0.43706	0.89943	0.48593	2.05790
56	0.43733	0.89930	0.48629	2.05637
57	0.43759	0.89918	0.48665	2.05485
58	0.43785	0.89905	0.48701	2.05333
59	0.43811	0.89892	0.48737	2.05182
60	0.43837	0.89879	0.48773	2.05030

Cos	Sin	Cot	Tan	Minutes
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26 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.43837	0.89879	0.48773	2.05030	60
1	0.43863	0.89867	0.48809	2.04879	59
2	0.43889	0.89854	0.48845	2.04728	58
3	0.43916	0.89841	0.48881	2.04577	57
4	0.43942	0.89828	0.48917	2.04426	56
5	0.43968	0.89816	0.48953	2.04276	55
6	0.43994	0.89803	0.48989	2.04125	54
7	0.44020	0.89790	0.49026	2.03975	53
8	0.44046	0.89777	0.49062	2.03825	52
9	0.44072	0.89764	0.49098	2.03675	51
10	0.44098	0.89752	0.49134	2.03526	50
11	0.44124	0.89739	0.49170	2.03376	49
12	0.44151	0.89726	0.49206	2.03227	48
13	0.44177	0.89713	0.49242	2.03078	47
14	0.44203	0.89700	0.49278	2.02929	46
15	0.44229	0.89687	0.49315	2.02780	45
16	0.44255	0.89674	0.49351	2.02631	44
17	0.44281	0.89662	0.49387	2.02483	43
18	0.44307	0.89649	0.49423	2.02335	42
19	0.44333	0.89636	0.49459	2.02187	41
20	0.44359	0.89623	0.49495	2.02039	40
21	0.44385	0.89610	0.49532	2.01891	39
22	0.44411	0.89597	0.49568	2.01743	38
23	0.44437	0.89584	0.49604	2.01596	37
24	0.44464	0.89571	0.49640	2.01449	36
25	0.44490	0.89558	0.49677	2.01302	35
26	0.44516	0.89545	0.49713	2.01155	34
27	0.44542	0.89532	0.49749	2.01008	33
28	0.44568	0.89519	0.49786	2.00862	32
29	0.44594	0.89506	0.49822	2.00715	31
30	0.44620	0.89493	0.49858	2.00569	30
31	0.44646	0.89480	0.49894	2.00423	29
32	0.44672	0.89467	0.49931	2.00277	28
33	0.44698	0.89454	0.49967	2.00131	27
34	0.44724	0.89441	0.50004	1.99986	26
35	0.44750	0.89428	0.50040	1.99841	25
36	0.44776	0.89415	0.50076	1.99695	24
37	0.44802	0.89402	0.50113	1.99550	23
38	0.44828	0.89389	0.50149	1.99406	22
39	0.44854	0.89376	0.50185	1.99261	21
40	0.44880	0.89363	0.50222	1.99116	20
41	0.44906	0.89350	0.50258	1.98972	19
42	0.44932	0.89337	0.50295	1.98828	18
43	0.44958	0.89324	0.50331	1.98684	17
44	0.44984	0.89311	0.50368	1.98540	16
45	0.45010	0.89298	0.50404	1.98396	15
46	0.45036	0.89285	0.50441	1.98253	14
47	0.45062	0.89272	0.50477	1.98110	13
48	0.45088	0.89259	0.50514	1.97966	12
49	0.45114	0.89245	0.50550	1.97823	11
50	0.45140	0.89232	0.50587	1.97681	10
51	0.45166	0.89219	0.50623	1.97538	9
52	0.45192	0.89206	0.50660	1.97395	8
53	0.45218	0.89193	0.50696	1.97253	7
54	0.45243	0.89180	0.50733	1.97111	6
55	0.45269	0.89167	0.50769	1.96969	5
56	0.45295	0.89153	0.50806	1.96827	4
57	0.45321	0.89140	0.50843	1.96685	3
58	0.45347	0.89127	0.50879	1.96544	2
59	0.45373	0.89114	0.50916	1.96402	1
60	0.45399	0.89101	0.50953	1.96261	0

Cos

Sin

Cot

Tan

Minutes

63 Degrees

27 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.45399	0.89101	0.50953	1.96261	60
1	0.45425	0.89087	0.50989	1.96120	59
2	0.45451	0.89074	0.51026	1.95979	58
3	0.45477	0.89061	0.51063	1.95838	57
4	0.45503	0.89048	0.51099	1.95698	56
5	0.45529	0.89035	0.51136	1.95557	55
6	0.45554	0.89021	0.51173	1.95417	54
7	0.45580	0.89008	0.51209	1.95277	53
8	0.45606	0.88995	0.51246	1.95137	52
9	0.45632	0.88981	0.51283	1.94997	51
10	0.45658	0.88968	0.51319	1.94858	50
11	0.45684	0.88955	0.51356	1.94718	49
12	0.45710	0.88942	0.51393	1.94579	48
13	0.45736	0.88928	0.51430	1.94440	47
14	0.45762	0.88915	0.51467	1.94301	46
15	0.45787	0.88902	0.51503	1.94162	45
16	0.45813	0.88888	0.51540	1.94023	44
17	0.45839	0.88875	0.51577	1.93885	43
18	0.45865	0.88862	0.51614	1.93746	42
19	0.45891	0.88848	0.51651	1.93608	41
20	0.45917	0.88835	0.51688	1.93470	40
21	0.45942	0.88822	0.51724	1.93332	39
22	0.45968	0.88808	0.51761	1.93195	38
23	0.45994	0.88795	0.51798	1.93057	37
24	0.46020	0.88782	0.51835	1.92920	36
25	0.46046	0.88768	0.51872	1.92782	35
26	0.46072	0.88755	0.51909	1.92645	34
27	0.46097	0.88741	0.51946	1.92508	33
28	0.46123	0.88728	0.51983	1.92371	32
29	0.46149	0.88715	0.52020	1.92235	31
30	0.46175	0.88701	0.52057	1.92098	30
31	0.46201	0.88688	0.52094	1.91962	29
32	0.46226	0.88674	0.52131	1.91826	28
33	0.46252	0.88661	0.52168	1.91690	27
34	0.46278	0.88647	0.52205	1.91554	26
35	0.46304	0.88634	0.52242	1.91418	25
36	0.46330	0.88620	0.52279	1.91282	24
37	0.46355	0.88607	0.52316	1.91147	23
38	0.46381	0.88593	0.52353	1.91012	22
39	0.46407	0.88580	0.52390	1.90876	21
40	0.46433	0.88566	0.52427	1.90741	20
41	0.46458	0.88553	0.52464	1.90607	19
42	0.46484	0.88539	0.52501	1.90472	18
43	0.46510	0.88526	0.52538	1.90337	17
44	0.46536	0.88512	0.52575	1.90203	16
45	0.46561	0.88499	0.52613	1.90069	15
46	0.46587	0.88485	0.52650	1.89935	14
47	0.46613	0.88472	0.52687	1.89801	13
48	0.46639	0.88458	0.52724	1.89667	12
49	0.46664	0.88445	0.52761	1.89533	11
50	0.46690	0.88431	0.52798	1.89400	10
51	0.46716	0.88417	0.52836	1.89266	9
52	0.46742	0.88404	0.52873	1.89133	8
53	0.46767	0.88390	0.52910	1.89000	7
54	0.46793	0.88377	0.52947	1.88867	6
55	0.46819	0.88363	0.52985	1.88734	5
56	0.46844	0.88349	0.53022	1.88602	4
57	0.46870	0.88336	0.53059	1.88469	3
58	0.46896	0.88322	0.53096	1.88337	2
59	0.46921	0.88308	0.53134	1.88205	1
60	0.46947	0.88295	0.53171	1.88073	0

Cos Sin Cot Tan Minutes

62 Degrees

	28	Degree			
Minutes	Sin	Cos	Tan	Cot	
0	0.46947	0.88295	0.53171	1.88073	60
1	0.46973	0.88281	0.53208	1.87941	59
2	0.46999	0.88267	0.53246	1.87809	58
3	0.47024	0.88254	0.53283	1.87677	57
4	0.47050	0.88240	0.53320	1.87546	56
5	0.47076	0.88226	0.53358	1.87415	55
6	0.47101	0.88213	0.53395	1.87283	54
7	0.47127	0.88199	0.53432	1.87152	53
8	0.47153	0.88185	0.53470	1.87021	52
9	0.47178	0.88172	0.53507	1.86891	51
10	0.47204	0.88158	0.53545	1.86760	50
11	0.47229	0.88144	0.53582	1.86630	49
12	0.47255	0.88130	0.53620	1.86499	48
13	0.47281	0.88117	0.53657	1.86369	47
14	0.47306	0.88103	0.53694	1.86239	46
15	0.47332	0.88089	0.53732	1.86109	45
16	0.47358	0.88075	0.53769	1.85979	44
17	0.47383	0.88062	0.53807	1.85850	43
18	0.47409	0.88048	0.53844	1.85720	42
19	0.47434	0.88034	0.53882	1.85591	41
20	0.47460	0.88020	0.53920	1.85462	40
21	0.47486	0.88006	0.53957	1.85333	39
22	0.47511	0.87993	0.53995	1.85204	38
23	0.47537	0.87979	0.54032	1.85075	37
24	0.47562	0.87965	0.54070	1.84946	36
25	0.47588	0.87951	0.54107	1.84818	35
26	0.47614	0.87937	0.54145	1.84689	34
27	0.47639	0.87923	0.54183	1.84561	33
28	0.47665	0.87909	0.54220	1.84433	32
29	0.47690	0.87896	0.54258	1.84305	31
30	0.47716	0.87882	0.54296	1.84177	30
31	0.47741	0.87868	0.54333	1.84049	29
32	0.47767	0.87854	0.54371	1.83922	28
33	0.47793	0.87840	0.54409	1.83794	27
34	0.47818	0.87826	0.54446	1.83667	26
35	0.47844	0.87812	0.54484	1.83540	25
36	0.47869	0.87798	0.54522	1.83413	24
37	0.47895	0.87784	0.54560	1.83286	23
38	0.47920	0.87770	0.54597	1.83159	22
39	0.47946	0.87756	0.54635	1.83033	21
40	0.47971	0.87743	0.54673	1.82906	20
41	0.47997	0.87729	0.54711	1.82780	19
42	0.48022	0.87715	0.54748	1.82654	18
43	0.48048	0.87701	0.54786	1.82528	17
44	0.48073	0.87687	0.54824	1.82402	16
45	0.48099	0.87673	0.54862	1.82276	15
46	0.48124	0.87659	0.54900	1.82150	14
47	0.48150	0.87645	0.54938	1.82025	13
48	0.48175	0.87631	0.54975	1.81899	12
49	0.48201	0.87617	0.55013	1.81774	11
50	0.48226	0.87603	0.55051	1.81649	10
51	0.48252	0.87589	0.55089	1.81524	9
52	0.48277	0.87575	0.55127	1.81399	8
53	0.48303	0.87561	0.55165	1.81274	7
54	0.48328	0.87546	0.55203	1.81150	6
55	0.48354	0.87532	0.55241	1.81025	5
56	0.48379	0.87518	0.55279	1.80901	4
57	0.48405	0.87504	0.55317	1.80777	3
58	0.48430	0.87490	0.55355	1.80653	2
59	0.48456	0.87476	0.55393	1.80529	1
60	0.48481	0.87462	0.55431	1.80405	0

Cos	Sin	Cot	Tan	Minutes
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29 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.48481	0.87462	0.55431	1.80405	60
1	0.48506	0.87448	0.55469	1.80281	59
2	0.48532	0.87434	0.55507	1.80158	58
3	0.48557	0.87420	0.55545	1.80034	57
4	0.48583	0.87406	0.55583	1.79911	56
5	0.48608	0.87391	0.55621	1.79788	55
6	0.48634	0.87377	0.55659	1.79665	54
7	0.48659	0.87363	0.55697	1.79542	53
8	0.48684	0.87349	0.55736	1.79419	52
9	0.48710	0.87335	0.55774	1.79296	51
10	0.48735	0.87321	0.55812	1.79174	50
11	0.48761	0.87306	0.55850	1.79051	49
12	0.48786	0.87292	0.55888	1.78929	48
13	0.48811	0.87278	0.55926	1.78807	47
14	0.48837	0.87264	0.55964	1.78685	46
15	0.48862	0.87250	0.56003	1.78563	45
16	0.48888	0.87235	0.56041	1.78441	44
17	0.48913	0.87221	0.56079	1.78319	43
18	0.48938	0.87207	0.56117	1.78198	42
19	0.48964	0.87193	0.56156	1.78077	41
20	0.48989	0.87178	0.56194	1.77955	40
21	0.49014	0.87164	0.56232	1.77834	39
22	0.49040	0.87150	0.56270	1.77713	38
23	0.49065	0.87136	0.56309	1.77592	37
24	0.49090	0.87121	0.56347	1.77471	36
25	0.49115	0.87107	0.56385	1.77351	35
26	0.49141	0.87093	0.56424	1.77230	34
27	0.49166	0.87079	0.56462	1.77110	33
28	0.49192	0.87064	0.56501	1.76990	32
29	0.49217	0.87050	0.56539	1.76869	31
30	0.49242	0.87036	0.56577	1.76749	30
31	0.49268	0.87021	0.56616	1.76629	29
32	0.49293	0.87007	0.56654	1.76510	28
33	0.49318	0.86993	0.56693	1.76390	27
34	0.49344	0.86978	0.56731	1.76271	26
35	0.49369	0.86964	0.56769	1.76151	25
36	0.49394	0.86949	0.56808	1.76032	24
37	0.49419	0.86935	0.56846	1.75913	23
38	0.49445	0.86921	0.56885	1.75794	22
39	0.49470	0.86906	0.56923	1.75675	21
40	0.49495	0.86892	0.56962	1.75556	20
41	0.49521	0.86878	0.57000	1.75437	19
42	0.49546	0.86863	0.57039	1.75319	18
43	0.49571	0.86849	0.57078	1.75200	17
44	0.49596	0.86834	0.57116	1.75082	16
45	0.49622	0.86820	0.57155	1.74964	15
46	0.49647	0.86805	0.57193	1.74846	14
47	0.49672	0.86791	0.57232	1.74728	13
48	0.49697	0.86777	0.57271	1.74610	12
49	0.49723	0.86762	0.57309	1.74492	11
50	0.49748	0.86748	0.57348	1.74375	10
51	0.49773	0.86733	0.57386	1.74257	9
52	0.49798	0.86719	0.57425	1.74140	8
53	0.49824	0.86704	0.57464	1.74022	7
54	0.49849	0.86690	0.57503	1.73905	6
55	0.49874	0.86675	0.57541	1.73788	5
56	0.49899	0.86661	0.57580	1.73671	4
57	0.49924	0.86646	0.57619	1.73555	3
58	0.49950	0.86632	0.57657	1.73438	2
59	0.49975	0.86617	0.57696	1.73321	1
60	0.50000	0.86603	0.57735	1.73205	0

Cos Sin Cot Tan Minutes

60 Degrees

	30	Degree				
Minutes		Sin	Cos	Tan	Cot	Minutes
0	0	0.86603	0.57735	1.53205	0.57735	60
1	1	0.86588	0.57774	1.73089	0.57774	59
2	2	0.86573	0.57813	1.72973	0.57813	58
3	3	0.86559	0.57851	1.72857	0.57851	57
4	4	0.86544	0.57889	1.72741	0.57889	56
5	5	0.86530	0.57929	1.72625	0.57929	55
6	6	0.86515	0.57968	1.72509	0.57968	54
7	7	0.86501	0.58007	1.72393	0.58007	53
8	8	0.86486	0.58046	1.72278	0.58046	52
9	9	0.86471	0.58085	1.72163	0.58085	51
10	10	0.86457	0.58124	1.72047	0.58124	50
11	11	0.86442	0.58162	1.71932	0.58162	49
12	12	0.86427	0.58201	1.71817	0.58201	48
13	13	0.86413	0.58240	1.71702	0.58240	47
14	14	0.86388	0.58279	1.71588	0.58279	46
15	15	0.86384	0.58318	1.71473	0.58318	45
16	16	0.86369	0.58357	1.71358	0.58357	44
17	17	0.86354	0.58396	1.71244	0.58396	43
18	18	0.86340	0.58435	1.71129	0.58435	42
19	19	0.86325	0.58474	1.71015	0.58474	41
20	20	0.86310	0.58513	1.70901	0.58513	40
21	21	0.86295	0.58552	1.70787	0.58552	39
22	22	0.86281	0.58591	1.70673	0.58591	38
23	23	0.86266	0.58631	1.70560	0.58631	37
24	24	0.86251	0.58670	1.70446	0.58670	36
25	25	0.86237	0.58709	1.70332	0.58709	35
26	26	0.86222	0.58748	1.70219	0.58748	34
27	27	0.86207	0.58787	1.70106	0.58787	33
28	28	0.86192	0.58826	1.69992	0.58826	32
29	29	0.86178	0.58865	1.69879	0.58865	31
30	30	0.86163	0.58905	1.69766	0.58905	30
31	31	0.86148	0.58944	1.69653	0.58944	29
32	32	0.86133	0.58983	1.69541	0.58983	28
33	33	0.86119	0.59022	1.69428	0.59022	27
34	34	0.86104	0.59061	1.69316	0.59061	26
35	35	0.86089	0.59101	1.69203	0.59101	25
36	36	0.86074	0.59140	1.69091	0.59140	24
37	37	0.86059	0.59179	1.68979	0.59179	23
38	38	0.86045	0.59218	1.68866	0.59218	22
39	39	0.86030	0.59258	1.68754	0.59258	21
40	40	0.86015	0.59297	1.68643	0.59297	20
41	41	0.86000	0.59336	1.68531	0.59336	19
42	42	0.85985	0.59376	1.68419	0.59376	18
43	43	0.85970	0.59415	1.68306	0.59415	17
44	44	0.85956	0.59454	1.68196	0.59454	16
45	45	0.85941	0.59494	1.68085	0.59494	15
46	46	0.85926	0.59533	1.67974	0.59533	14
47	47	0.85911	0.59573	1.67863	0.59573	13
48	48	0.85896	0.59612	1.67752	0.59612	12
49	49	0.85881	0.59651	1.67641	0.59651	11
50	50	0.85866	0.59691	1.67530	0.59691	10
51	51	0.85851	0.59730	1.67419	0.59730	9
52	52	0.85836	0.59770	1.67309	0.59770	8
53	53	0.85821	0.59809	1.67198	0.59809	7
54	54	0.85806	0.59849	1.67088	0.59849	6
55	55	0.85792	0.59888	1.66978	0.59888	5
56	56	0.85777	0.59928	1.66867	0.59928	4
57	57	0.85762	0.59967	1.66757	0.59967	3
58	58	0.85747	0.60007	1.66647	0.60007	2
59	59	0.85732	0.60046	1.66538	0.60046	1
60	60	0.85717	0.60086	1.66428	0.60086	0

Cos Sin Cot Tan
59 Degree

31 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.51504	0.85717	0.60086	1.66426	60
1	0.51529	0.85702	0.60126	1.66318	59
2	0.51554	0.85687	0.60165	1.66209	58
3	0.51579	0.85672	0.60205	1.66099	57
4	0.51604	0.85657	0.60245	1.65990	56
5	0.51628	0.85642	0.60284	1.65881	55
6	0.51653	0.85627	0.60324	1.65772	54
7	0.51678	0.85612	0.60364	1.65663	53
8	0.51703	0.85597	0.60403	1.65554	52
9	0.51728	0.85582	0.60443	1.65445	51
10	0.51753	0.85567	0.60483	1.65337	50
11	0.51778	0.85551	0.60522	1.65228	49
12	0.51803	0.85536	0.60562	1.65120	48
13	0.51828	0.85521	0.60602	1.65011	47
14	0.51852	0.85506	0.60642	1.64903	46
15	0.51877	0.85491	0.60681	1.64795	45
16	0.51902	0.85476	0.60721	1.64687	44
17	0.51927	0.85461	0.60761	1.64579	43
18	0.51952	0.85446	0.60801	1.64471	42
19	0.51977	0.85431	0.60841	1.64363	41
20	0.52002	0.85416	0.60881	1.64256	40
21	0.52026	0.85401	0.60921	1.64148	39
22	0.52051	0.85385	0.60960	1.64041	38
23	0.52076	0.85370	0.61000	1.63934	37
24	0.52101	0.85355	0.61040	1.63826	36
25	0.52126	0.85340	0.61080	1.63719	35
26	0.52151	0.85325	0.61120	1.63612	34
27	0.52175	0.85310	0.61160	1.63505	33
28	0.52200	0.85294	0.61200	1.63398	32
29	0.52225	0.85279	0.61240	1.63292	31
30	0.52250	0.85264	0.61280	1.63185	30
31	0.52275	0.85249	0.61320	1.63079	29
32	0.52299	0.85234	0.61360	1.62972	28
33	0.52324	0.85218	0.61400	1.62866	27
34	0.52349	0.85203	0.61440	1.62760	26
35	0.52374	0.85188	0.61480	1.62654	25
36	0.52399	0.85173	0.61520	1.62548	24
37	0.52423	0.85157	0.61561	1.62442	23
38	0.52448	0.85142	0.61601	1.62336	22
39	0.52473	0.85127	0.61641	1.62230	21
40	0.52498	0.85112	0.61681	1.62125	20
41	0.52522	0.85096	0.61721	1.62019	19
42	0.52547	0.85081	0.61761	1.61914	18
43	0.52572	0.85066	0.61801	1.61808	17
44	0.52597	0.85051	0.61842	1.61703	16
45	0.52621	0.85035	0.61882	1.61598	15
46	0.52646	0.85020	0.61922	1.61493	14
47	0.52671	0.85005	0.61962	1.61388	13
48	0.52696	0.84989	0.62003	1.61283	12
49	0.52720	0.84974	0.62043	1.61179	11
50	0.52745	0.84959	0.62083	1.61074	10
51	0.52770	0.84943	0.62124	1.60970	9
52	0.52794	0.84928	0.62164	1.60865	8
53	0.52819	0.84913	0.62204	1.60761	7
54	0.52844	0.84897	0.62245	1.60657	6
55	0.52869	0.84882	0.62285	1.60553	5
56	0.52893	0.84866	0.62325	1.60449	4
57	0.52918	0.84851	0.62366	1.60345	3
58	0.52943	0.84836	0.62406	1.60241	2
59	0.52967	0.84820	0.62446	1.60137	1
60	0.52992	0.84805	0.62487	1.60033	0

Cos Sin Cot Tan Minutes

32 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.52992	0.84805	0.62487	1.60033
1	0.53017	0.84789	0.62527	1.59930
2	0.53041	0.84774	0.62568	1.59826
3	0.53066	0.84759	0.62608	1.59723
4	0.53091	0.84743	0.62649	1.59620
5	0.53115	0.84728	0.62689	1.59517
6	0.53140	0.84712	0.62730	1.59414
7	0.53164	0.84697	0.62770	1.59311
8	0.53189	0.84681	0.62811	1.59208
9	0.53214	0.84666	0.62852	1.59105
10	0.53238	0.84650	0.62892	1.59002
11	0.53263	0.84635	0.62933	1.58900
12	0.53288	0.84619	0.62973	1.58797
13	0.53312	0.84604	0.63014	1.58695
14	0.53337	0.84588	0.63055	1.58593
15	0.53361	0.84573	0.63095	1.58490
16	0.53386	0.84557	0.63136	1.58388
17	0.53411	0.84542	0.63177	1.58286
18	0.53435	0.84526	0.63217	1.58184
19	0.53460	0.84511	0.63258	1.58083
20	0.53484	0.84495	0.63299	1.57981
21	0.53509	0.84480	0.63340	1.57879
22	0.53534	0.84464	0.63380	1.57778
23	0.53558	0.84448	0.63421	1.57676
24	0.53583	0.84433	0.63462	1.57575
25	0.53607	0.84417	0.63503	1.57474
26	0.53632	0.84402	0.63544	1.57372
27	0.53656	0.84386	0.63584	1.57271
28	0.53681	0.84370	0.63625	1.57170
29	0.53705	0.84355	0.63666	1.57069
30	0.53730	0.84339	0.63707	1.56969
31	0.53754	0.84324	0.63748	1.56868
32	0.53779	0.84308	0.63789	1.56767
33	0.53804	0.84292	0.63830	1.56667
34	0.53828	0.84277	0.63871	1.56566
35	0.53853	0.84261	0.63912	1.56466
36	0.53877	0.84245	0.63953	1.56366
37	0.53902	0.84230	0.63994	1.56265
38	0.53926	0.84214	0.64035	1.56165
39	0.53951	0.84198	0.64076	1.56065
40	0.53975	0.84182	0.64117	1.55966
41	0.54000	0.84167	0.64158	1.55866
42	0.54024	0.84151	0.64199	1.55766
43	0.54049	0.84135	0.64240	1.55666
44	0.54073	0.84120	0.64281	1.55567
45	0.54097	0.84104	0.64322	1.55467
46	0.54122	0.84088	0.64363	1.55368
47	0.54146	0.84072	0.64404	1.55269
48	0.54171	0.84057	0.64446	1.55170
49	0.54195	0.84041	0.64487	1.55071
50	0.54220	0.84025	0.64528	1.54972
51	0.54244	0.84009	0.64569	1.54873
52	0.54269	0.83994	0.64610	1.54774
53	0.54293	0.83978	0.64652	1.54675
54	0.54317	0.83962	0.64693	1.54576
55	0.54342	0.83946	0.64734	1.54478
56	0.54366	0.83930	0.64775	1.54379
57	0.54391	0.83915	0.64817	1.54281
58	0.54415	0.83899	0.64858	1.54183
59	0.54440	0.83883	0.64899	1.54085
60	0.54464	0.83867	0.64941	1.53986

Cos	Sin	Cot	Tan	Minutes
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57 Degrees

	33 Degree				
Minutes	Sin	Cos	Tan	Cot	
0	0.54464	0.83867	0.64941	1.53986	89
1	0.54488	0.83851	0.64982	1.53988	88
2	0.54513	0.83835	0.65024	1.53791	86
3	0.54537	0.83819	0.65065	1.53693	84
4	0.54561	0.83804	0.65106	1.53595	82
5	0.54586	0.83788	0.65148	1.53497	80
6	0.54610	0.83772	0.65189	1.53400	78
7	0.54635	0.83756	0.65231	1.53302	76
8	0.54659	0.83740	0.65272	1.53205	74
9	0.54683	0.83724	0.65314	1.53107	72
10	0.54708	0.83708	0.65355	1.53010	70
11	0.54732	0.83692	0.65397	1.52913	68
12	0.54756	0.83676	0.65438	1.52816	66
13	0.54781	0.83660	0.65480	1.52719	64
14	0.54805	0.83645	0.65521	1.52622	62
15	0.54829	0.83629	0.65563	1.52525	60
16	0.54854	0.83613	0.65604	1.52429	58
17	0.54878	0.83597	0.65646	1.52332	56
18	0.54902	0.83581	0.65688	1.52235	54
19	0.54927	0.83565	0.65729	1.52138	52
20	0.54951	0.83549	0.65771	1.52043	50
21	0.54975	0.83533	0.65813	1.51946	48
22	0.54999	0.83517	0.65854	1.51850	46
23	0.55024	0.83501	0.65896	1.51754	44
24	0.55048	0.83485	0.65938	1.51658	42
25	0.55072	0.83469	0.65980	1.51562	40
26	0.55097	0.83453	0.66021	1.51466	38
27	0.55121	0.83437	0.66063	1.51370	36
28	0.55145	0.83421	0.66105	1.51275	34
29	0.55169	0.83405	0.66147	1.51179	32
30	0.55194	0.83389	0.66189	1.51084	30
31	0.55218	0.83373	0.66230	1.50988	29
32	0.55242	0.83356	0.66272	1.50893	28
33	0.55266	0.83340	0.66314	1.50797	27
34	0.55291	0.83324	0.66356	1.50702	26
35	0.55315	0.83308	0.66398	1.50607	25
36	0.55339	0.83292	0.66440	1.50512	24
37	0.55363	0.83276	0.66482	1.50417	23
38	0.55388	0.83260	0.66524	1.50322	22
39	0.55412	0.83244	0.66566	1.50228	21
40	0.55436	0.83228	0.66608	1.50133	20
41	0.55460	0.83212	0.66650	1.50038	19
42	0.55484	0.83195	0.66692	1.49944	18
43	0.55509	0.83179	0.66734	1.49849	17
44	0.55533	0.83163	0.66776	1.49755	16
45	0.55557	0.83147	0.66818	1.49661	15
46	0.55581	0.83131	0.66860	1.49566	14
47	0.55605	0.83115	0.66902	1.49472	13
48	0.55630	0.83098	0.66944	1.49378	12
49	0.55654	0.83082	0.66986	1.49284	11
50	0.55678	0.83066	0.67028	1.49190	10
51	0.55702	0.83050	0.67071	1.49097	9
52	0.55726	0.83034	0.67113	1.49003	8
53	0.55750	0.83017	0.67155	1.48909	7
54	0.55775	0.83001	0.67197	1.48816	6
55	0.55799	0.82985	0.67239	1.48722	5
56	0.55823	0.82969	0.67282	1.48629	4
57	0.55847	0.82953	0.67324	1.48536	3
58	0.55871	0.82936	0.67366	1.48442	2
59	0.55895	0.82920	0.67409	1.48348	1
60	0.55919	0.82904	0.67451	1.48256	0

	Cos	Sin	Cot	Tan	Minutes

34 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.55919	0.82904	0.67451	1.48256	60
1	0.55943	0.82887	0.67493	1.48163	59
2	0.55968	0.82871	0.67536	1.48070	58
3	0.55992	0.82855	0.67578	1.47977	57
4	0.56016	0.82839	0.67620	1.47885	56
5	0.56040	0.82822	0.67663	1.47792	55
6	0.56064	0.82806	0.67705	1.47699	54
7	0.56088	0.82790	0.67748	1.47607	53
8	0.56112	0.82773	0.67790	1.47514	52
9	0.56136	0.82757	0.67832	1.47422	51
10	0.56160	0.82741	0.67875	1.47330	50
11	0.56184	0.82724	0.67917	1.47238	49
12	0.56208	0.82708	0.67960	1.47146	48
13	0.56232	0.82692	0.68002	1.47053	47
14	0.56256	0.82675	0.68045	1.46962	46
15	0.56280	0.82659	0.68088	1.46870	45
16	0.56305	0.82643	0.68130	1.46778	44
17	0.56329	0.82626	0.68173	1.46686	43
18	0.56353	0.82610	0.68215	1.46595	42
19	0.56377	0.82593	0.68258	1.46503	41
20	0.56401	0.82577	0.68301	1.46411	40
21	0.56425	0.82561	0.68343	1.46320	39
22	0.56449	0.82544	0.68386	1.46229	38
23	0.56473	0.82528	0.68429	1.46137	37
24	0.56497	0.82511	0.68471	1.46046	36
25	0.56521	0.82495	0.68514	1.45955	35
26	0.56545	0.82478	0.68557	1.45864	34
27	0.56569	0.82462	0.68600	1.45773	33
28	0.56593	0.82446	0.68642	1.45682	32
29	0.56617	0.82429	0.68685	1.45592	31
30	0.56641	0.82413	0.68728	1.45501	30
31	0.56665	0.82396	0.68771	1.45410	29
32	0.56689	0.82380	0.68814	1.45320	28
33	0.56713	0.82363	0.68857	1.45229	27
34	0.56736	0.82347	0.68900	1.45139	26
35	0.56760	0.82330	0.68942	1.45049	25
36	0.56784	0.82314	0.68985	1.44958	24
37	0.56808	0.82297	0.69028	1.44868	23
38	0.56832	0.82281	0.69071	1.44778	22
39	0.56856	0.82264	0.69114	1.44688	21
40	0.56880	0.82248	0.69157	1.44598	20
41	0.56904	0.82231	0.69200	1.44508	19
42	0.56928	0.82214	0.69243	1.44418	18
43	0.56952	0.82198	0.69286	1.44329	17
44	0.56976	0.82181	0.69329	1.44239	16
45	0.57000	0.82165	0.69372	1.44149	15
46	0.57024	0.82148	0.69416	1.44060	14
47	0.57047	0.82132	0.69459	1.43970	13
48	0.57071	0.82115	0.69502	1.43881	12
49	0.57095	0.82098	0.69545	1.43792	11
50	0.57119	0.82082	0.69588	1.43703	10
51	0.57143	0.82065	0.69631	1.43614	9
52	0.57167	0.82048	0.69675	1.43525	8
53	0.57191	0.82032	0.69718	1.43436	7
54	0.57215	0.82015	0.69761	1.43347	6
55	0.57238	0.81999	0.69804	1.43258	5
56	0.57262	0.81982	0.69847	1.43169	4
57	0.57286	0.81965	0.69891	1.43080	3
58	0.57310	0.81949	0.69934	1.42992	2
59	0.57334	0.81932	0.69977	1.42903	1
60	0.57358	0.81915	0.70021	1.42815	0

Cos

Sin

Cot

Tan

Minutes

55 Degrees

35 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.57358	0.81915	0.70021	1.42815
1	0.57381	0.81899	0.70064	1.42726
2	0.57405	0.81882	0.70107	1.42638
3	0.57429	0.81865	0.70151	1.42550
4	0.57453	0.81848	0.70194	1.42462
5	0.57477	0.81832	0.70238	1.42374
6	0.57501	0.81815	0.70281	1.42286
7	0.57524	0.81798	0.70325	1.42198
8	0.57548	0.81782	0.70368	1.42110
9	0.57572	0.81765	0.70412	1.42022
10	0.57596	0.81748	0.70455	1.41934
11	0.57619	0.81731	0.70499	1.41847
12	0.57643	0.81714	0.70542	1.41759
13	0.57667	0.81698	0.70586	1.41672
14	0.57691	0.81681	0.70629	1.41584
15	0.57715	0.81664	0.70673	1.41497
16	0.57738	0.81647	0.70717	1.41409
17	0.57762	0.81631	0.70760	1.41322
18	0.57786	0.81614	0.70804	1.41235
19	0.57810	0.81597	0.70848	1.41148
20	0.57833	0.81580	0.70891	1.41061
21	0.57857	0.81563	0.70935	1.40974
22	0.57881	0.81546	0.70979	1.40887
23	0.57904	0.81530	0.71023	1.40800
24	0.57928	0.81513	0.71066	1.40714
25	0.57952	0.81496	0.71110	1.40627
26	0.57976	0.81479	0.71154	1.40540
27	0.57999	0.81462	0.71198	1.40454
28	0.58023	0.81445	0.71242	1.40367
29	0.58047	0.81428	0.71285	1.40281
30	0.58070	0.81412	0.71329	1.40195
31	0.58094	0.81395	0.71373	1.40109
32	0.58118	0.81378	0.71417	1.40022
33	0.58141	0.81361	0.71461	1.39936
34	0.58165	0.81344	0.71505	1.39850
35	0.58189	0.81327	0.71549	1.39764
36	0.58212	0.81310	0.71593	1.39679
37	0.58236	0.81293	0.71637	1.39593
38	0.58260	0.81276	0.71681	1.39507
39	0.58283	0.81259	0.71725	1.39421
40	0.58307	0.81242	0.71769	1.39336
41	0.58330	0.81225	0.71813	1.39250
42	0.58354	0.81208	0.71857	1.39165
43	0.58378	0.81191	0.71901	1.39079
44	0.58401	0.81174	0.71946	1.38994
45	0.58425	0.81157	0.71990	1.38909
46	0.58449	0.81140	0.72034	1.38824
47	0.58472	0.81123	0.72078	1.38738
48	0.58496	0.81106	0.72122	1.38653
49	0.58519	0.81089	0.72167	1.38568
50	0.58543	0.81072	0.72211	1.38484
51	0.58567	0.81055	0.72255	1.38399
52	0.58590	0.81038	0.72299	1.38314
53	0.58614	0.81021	0.72344	1.38229
54	0.58637	0.81004	0.72388	1.38145
55	0.58661	0.80987	0.72432	1.38060
56	0.58684	0.80970	0.72477	1.37976
57	0.58708	0.80953	0.72521	1.37891
58	0.58731	0.80936	0.72565	1.37807
59	0.58755	0.80919	0.72610	1.37722
60	0.58779	0.80902	0.72654	1.37638

Cos	Sin	Cot	Tan	Minutes
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36 Degrees

Minutes	Sin	Cos	Tan	Cot	Minutes
0	0.58779	0.80902	0.72654	1.37638	60
1	0.58802	0.80885	0.72699	1.37554	59
2	0.58826	0.80867	0.72743	1.37470	58
3	0.58849	0.80850	0.72788	1.37386	57
4	0.58873	0.80833	0.72832	1.37302	56
5	0.58896	0.80816	0.72877	1.37218	55
6	0.58920	0.80799	0.72921	1.37134	54
7	0.58943	0.80782	0.72966	1.37050	53
8	0.58967	0.80765	0.73010	1.36967	52
9	0.58990	0.80748	0.73055	1.36883	51
10	0.59014	0.80730	0.73100	1.36800	50
11	0.59037	0.80713	0.73144	1.36716	49
12	0.59061	0.80696	0.73189	1.36633	48
13	0.59084	0.80679	0.73234	1.36549	47
14	0.59108	0.80662	0.73278	1.36466	46
15	0.59131	0.80644	0.73323	1.36383	45
16	0.59154	0.80627	0.73368	1.36300	44
17	0.59178	0.80610	0.73413	1.36217	43
18	0.59201	0.80593	0.73457	1.36134	42
19	0.59225	0.80576	0.73502	1.36051	41
20	0.59248	0.80558	0.73547	1.35968	40
21	0.59272	0.80541	0.73592	1.35885	39
22	0.59295	0.80524	0.73637	1.35802	38
23	0.59318	0.80507	0.73681	1.35719	37
24	0.59342	0.80489	0.73726	1.35637	36
25	0.59365	0.80472	0.73771	1.35554	35
26	0.59389	0.80455	0.73816	1.35472	34
27	0.59412	0.80438	0.73861	1.35389	33
28	0.59436	0.80420	0.73906	1.35307	32
29	0.59459	0.80403	0.73951	1.35224	31
30	0.59482	0.80386	0.73996	1.35142	30
31	0.59506	0.80368	0.74041	1.35060	29
32	0.59529	0.80351	0.74086	1.34978	28
33	0.59552	0.80334	0.74131	1.34896	27
34	0.59576	0.80316	0.74176	1.34814	26
35	0.59599	0.80299	0.74221	1.34732	25
36	0.59622	0.80282	0.74267	1.34650	24
37	0.59646	0.80264	0.74312	1.34568	23
38	0.59669	0.80247	0.74357	1.34487	22
39	0.59693	0.80230	0.74402	1.34405	21
40	0.59716	0.80212	0.74447	1.34323	20
41	0.59739	0.80195	0.74492	1.34242	19
42	0.59763	0.80178	0.74538	1.34160	18
43	0.59786	0.80160	0.74583	1.34079	17
44	0.59809	0.80143	0.74628	1.33998	16
45	0.59832	0.80125	0.74674	1.33916	15
46	0.59856	0.80108	0.74719	1.33835	14
47	0.59879	0.80091	0.74764	1.33754	13
48	0.59902	0.80073	0.74810	1.33673	12
49	0.59926	0.80056	0.74855	1.33592	11
50	0.59949	0.80038	0.74900	1.33511	10
51	0.59972	0.80021	0.74946	1.33430	9
52	0.59995	0.80003	0.74991	1.33349	8
53	0.60019	0.79986	0.75037	1.33268	7
54	0.60042	0.79968	0.75082	1.33187	6
55	0.60065	0.79951	0.75128	1.33107	5
56	0.60089	0.79934	0.75173	1.33026	4
57	0.60112	0.79916	0.75219	1.32946	3
58	0.60135	0.79899	0.75264	1.32865	2
59	0.60158	0.79881	0.75310	1.32785	1
60	0.60182	0.79864	0.75355	1.32704	0

Cos

Sin

Cot

Tan

Minutes

53 Degrees

37 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.60182	0.79864	0.75355	1.32704	60
1	0.60205	0.79846	0.75401	1.32624	59
2	0.60228	0.79829	0.75447	1.32544	58
3	0.60251	0.79811	0.75492	1.32464	57
4	0.60274	0.79793	0.75538	1.32384	56
5	0.60298	0.79776	0.75584	1.32304	55
6	0.60321	0.79758	0.75629	1.32224	54
7	0.60344	0.79741	0.75675	1.32144	53
8	0.60367	0.79723	0.75721	1.32064	52
9	0.60390	0.79706	0.75767	1.31984	51
10	0.60414	0.79688	0.75812	1.31904	50
11	0.60437	0.79671	0.75858	1.31825	49
12	0.60460	0.79653	0.75904	1.31745	48
13	0.60483	0.79635	0.75950	1.31666	47
14	0.60506	0.79618	0.75996	1.31586	46
15	0.60529	0.79600	0.76042	1.31507	45
16	0.60553	0.79583	0.76088	1.31427	44
17	0.60576	0.79565	0.76134	1.31348	43
18	0.60599	0.79547	0.76180	1.31269	42
19	0.60622	0.79530	0.76226	1.31190	41
20	0.60645	0.79512	0.76272	1.31110	40
21	0.60668	0.79494	0.76318	1.31031	39
22	0.60691	0.79477	0.76364	1.30952	38
23	0.60714	0.79459	0.76410	1.30873	37
24	0.60738	0.79441	0.76456	1.30795	36
25	0.60761	0.79424	0.76502	1.30716	35
26	0.60784	0.79406	0.76548	1.30637	34
27	0.60807	0.79388	0.76594	1.30558	33
28	0.60830	0.79371	0.76640	1.30480	32
29	0.60853	0.79353	0.76686	1.30401	31
30	0.60876	0.79335	0.76733	1.30323	30
31	0.60899	0.79318	0.76779	1.30244	29
32	0.60922	0.79300	0.76825	1.30166	28
33	0.60945	0.79282	0.76871	1.30087	27
34	0.60968	0.79264	0.76918	1.30009	26
35	0.60991	0.79247	0.76964	1.29931	25
36	0.61015	0.79229	0.77010	1.29853	24
37	0.61038	0.79211	0.77057	1.29775	23
38	0.61061	0.79193	0.77103	1.29696	22
39	0.61084	0.79176	0.77149	1.29618	21
40	0.61107	0.79158	0.77196	1.29541	20
41	0.61130	0.79140	0.77242	1.29463	19
42	0.61153	0.79122	0.77289	1.29385	18
43	0.61176	0.79105	0.77335	1.29307	17
44	0.61199	0.79087	0.77382	1.29229	16
45	0.61222	0.79069	0.77428	1.29152	15
46	0.61245	0.79051	0.77475	1.29074	14
47	0.61268	0.79033	0.77521	1.28997	13
48	0.61291	0.79016	0.77568	1.28919	12
49	0.61314	0.78998	0.77615	1.28842	11
50	0.61337	0.78980	0.77661	1.28764	10
51	0.61360	0.78962	0.77708	1.28687	9
52	0.61383	0.78944	0.77754	1.28610	8
53	0.61406	0.78926	0.77801	1.28533	7
54	0.61429	0.78908	0.77848	1.28456	6
55	0.61451	0.78891	0.77895	1.28379	5
56	0.61474	0.78873	0.77941	1.28302	4
57	0.61497	0.78855	0.77988	1.28225	3
58	0.61520	0.78837	0.78035	1.28148	2
59	0.61543	0.78819	0.78082	1.28071	1
60	0.61566	0.78801	0.78129	1.27994	0

Cos

Sin

Cot

Tan

Minutes

	Minutes	30	Degrees			
		Sin	Cos	Tan	Cot	
	0	0.61566	0.78801	0.78129	1.27994	60
	1	0.61569	0.78783	0.78175	1.27917	59
	2	0.61612	0.78765	0.78222	1.27841	58
	3	0.61635	0.78747	0.78269	1.27764	57
	4	0.61658	0.78729	0.78316	1.27688	56
	5	0.61681	0.78711	0.78363	1.27611	55
	6	0.61704	0.78694	0.78410	1.27535	54
	7	0.61726	0.78676	0.78457	1.27458	53
	8	0.61749	0.78658	0.78504	1.27382	52
	9	0.61772	0.78640	0.78551	1.27306	51
	10	0.61795	0.78622	0.78598	1.27230	50
	11	0.61818	0.78604	0.78645	1.27153	49
	12	0.61841	0.78586	0.78692	1.27077	48
	13	0.61864	0.78568	0.78739	1.27001	47
	14	0.61887	0.78550	0.78786	1.26925	46
	15	0.61909	0.78532	0.78834	1.26849	45
	16	0.61932	0.78514	0.78881	1.26774	44
	17	0.61955	0.78496	0.78928	1.26698	43
	18	0.61978	0.78478	0.78975	1.26622	42
	19	0.62001	0.78460	0.79022	1.26546	41
	20	0.62024	0.78442	0.79070	1.26471	40
	21	0.62046	0.78424	0.79117	1.26395	39
	22	0.62069	0.78405	0.79164	1.26319	38
	23	0.62092	0.78387	0.79212	1.26244	37
	24	0.62115	0.78369	0.79259	1.26169	36
	25	0.62138	0.78351	0.79306	1.26093	35
	26	0.62160	0.78333	0.79354	1.26018	34
	27	0.62183	0.78315	0.79401	1.25943	33
	28	0.62206	0.78297	0.79449	1.25867	32
	29	0.62229	0.78279	0.79496	1.25792	31
	30	0.62251	0.78261	0.79544	1.25717	30
	31	0.62274	0.78243	0.79591	1.25642	29
	32	0.62297	0.78225	0.79639	1.25567	28
	33	0.62320	0.78206	0.79686	1.25492	27
	34	0.62342	0.78188	0.79734	1.25417	26
	35	0.62365	0.78170	0.79781	1.25343	25
	36	0.62388	0.78152	0.79829	1.25268	24
	37	0.62411	0.78134	0.79877	1.25193	23
	38	0.62433	0.78116	0.79924	1.25118	22
	39	0.62456	0.78098	0.79972	1.25044	21
	40	0.62479	0.78080	0.80020	1.24969	20
	41	0.62502	0.78061	0.80067	1.24895	19
	42	0.62524	0.78043	0.80115	1.24820	18
	43	0.62547	0.78025	0.80163	1.24746	17
	44	0.62570	0.78007	0.80211	1.24672	16
	45	0.62592	0.77988	0.80258	1.24597	15
	46	0.62615	0.77970	0.80306	1.24523	14
	47	0.62638	0.77952	0.80354	1.24449	13
	48	0.62660	0.77934	0.80402	1.24375	12
	49	0.62683	0.77916	0.80450	1.24301	11
	50	0.62706	0.77897	0.80498	1.24227	10
	51	0.62728	0.77879	0.80546	1.24153	9
	52	0.62751	0.77861	0.80594	1.24079	8
	53	0.62774	0.77843	0.80642	1.24005	7
	54	0.62796	0.77824	0.80690	1.23931	6
	55	0.62819	0.77806	0.80738	1.23858	5
	56	0.62842	0.77788	0.80786	1.23784	4
	57	0.62864	0.77769	0.80834	1.23710	3
	58	0.62887	0.77751	0.80882	1.23637	2
	59	0.62909	0.77733	0.80930	1.23563	1
	60	0.62932	0.77715	0.80978	1.23490	0

	Cos	Sin	Cot	Tan	Minutes
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Minutes	Sin	Cos	Tan	Cot	Minutes
0	0.62932	0.77715	0.80978	1.23490	60
1	0.62955	0.77696	0.81027	1.23416	59
2	0.62977	0.77678	0.81075	1.23343	58
3	0.63000	0.77660	0.81123	1.23270	57
4	0.63022	0.77641	0.81171	1.23196	56
5	0.63045	0.77623	0.81220	1.23123	55
6	0.63068	0.77605	0.81268	1.23050	54
7	0.63090	0.77586	0.81316	1.22977	53
8	0.63113	0.77568	0.81364	1.22904	52
9	0.63135	0.77550	0.81413	1.22831	51
10	0.63158	0.77531	0.81461	1.22758	50
11	0.63180	0.77513	0.81510	1.22685	49
12	0.63203	0.77494	0.81558	1.22612	48
13	0.63225	0.77476	0.81606	1.22539	47
14	0.63248	0.77458	0.81655	1.22467	46
15	0.63271	0.77439	0.81703	1.22394	45
16	0.63293	0.77421	0.81752	1.22321	44
17	0.63316	0.77402	0.81800	1.22249	43
18	0.63338	0.77384	0.81849	1.22176	42
19	0.63361	0.77366	0.81898	1.22104	41
20	0.63383	0.77347	0.81946	1.22031	40
21	0.63406	0.77329	0.81995	1.21959	39
22	0.63428	0.77310	0.82044	1.21886	38
23	0.63451	0.77292	0.82092	1.21814	37
24	0.63473	0.77273	0.82141	1.21742	36
25	0.63496	0.77255	0.82190	1.21670	35
26	0.63518	0.77236	0.82238	1.21598	34
27	0.63540	0.77218	0.82287	1.21526	33
28	0.63563	0.77199	0.82336	1.21454	32
29	0.63585	0.77181	0.82385	1.21382	31
30	0.63608	0.77162	0.82434	1.21310	30
31	0.63630	0.77144	0.82483	1.21238	29
32	0.63653	0.77125	0.82531	1.21166	28
33	0.63675	0.77107	0.82580	1.21094	27
34	0.63698	0.77088	0.82629	1.21023	26
35	0.63720	0.77070	0.82678	1.20951	25
36	0.63742	0.77051	0.82727	1.20879	24
37	0.63765	0.77033	0.82776	1.20808	23
38	0.63787	0.77014	0.82825	1.20736	22
39	0.63810	0.76996	0.82874	1.20665	21
40	0.63832	0.76977	0.82923	1.20593	20
41	0.63854	0.76959	0.82972	1.20522	19
42	0.63877	0.76940	0.83022	1.20451	18
43	0.63899	0.76921	0.83071	1.20379	17
44	0.63922	0.76903	0.83120	1.20308	16
45	0.63944	0.76884	0.83169	1.20237	15
46	0.63966	0.76866	0.83218	1.20166	14
47	0.63989	0.76847	0.83268	1.20095	13
48	0.64011	0.76828	0.83317	1.20024	12
49	0.64033	0.76810	0.83366	1.19953	11
50	0.64056	0.76791	0.83415	1.19882	10
51	0.64078	0.76772	0.83465	1.19811	9
52	0.64100	0.76754	0.83514	1.19740	8
53	0.64123	0.76735	0.83564	1.19669	7
54	0.64145	0.76717	0.83613	1.19599	6
55	0.64167	0.76698	0.83662	1.19528	5
56	0.64190	0.76679	0.83712	1.19457	4
57	0.64212	0.76661	0.83761	1.19387	3
58	0.64234	0.76642	0.83811	1.19316	2
59	0.64256	0.76623	0.83860	1.19246	1
60	0.64279	0.76604	0.83910	1.19175	0

Cos

Sin

Cot

Tan

Minutes

50 Degrees

40 Degree				
Minutes	Sin	Cos	Tan	Cot
0	0.64279	0.76604	0.83910	1.19175
1	0.64301	0.76586	0.83960	1.19105
2	0.64323	0.76567	0.84009	1.19035
3	0.64346	0.76548	0.84059	1.18964
4	0.64368	0.76530	0.84108	1.18894
5	0.64390	0.76511	0.84158	1.18824
6	0.64412	0.76492	0.84208	1.18754
7	0.64435	0.76473	0.84258	1.18684
8	0.64457	0.76455	0.84307	1.18614
9	0.64479	0.76436	0.84357	1.18544
10	0.64501	0.76417	0.84407	1.18474
11	0.64524	0.76398	0.84457	1.18404
12	0.64546	0.76380	0.84507	1.18334
13	0.64568	0.76361	0.84556	1.18264
14	0.64590	0.76342	0.84606	1.18194
15	0.64612	0.76323	0.84656	1.18125
16	0.64635	0.76304	0.84706	1.18055
17	0.64657	0.76286	0.84756	1.17986
18	0.64679	0.76267	0.84806	1.17916
19	0.64701	0.76248	0.84856	1.17846
20	0.64723	0.76229	0.84906	1.17777
21	0.64746	0.76210	0.84956	1.17708
22	0.64768	0.76192	0.85006	1.17638
23	0.64790	0.76173	0.85057	1.17569
24	0.64812	0.76154	0.85107	1.17500
25	0.64834	0.76135	0.85157	1.17430
26	0.64856	0.76116	0.85207	1.17361
27	0.64878	0.76097	0.85257	1.17292
28	0.64901	0.76078	0.85308	1.17223
29	0.64923	0.76059	0.85358	1.17154
30	0.64945	0.76041	0.85408	1.17085
31	0.64967	0.76022	0.85458	1.17016
32	0.64989	0.76003	0.85509	1.16947
33	0.65011	0.75984	0.85559	1.16878
34	0.65033	0.75965	0.85609	1.16809
35	0.65055	0.75946	0.85660	1.16741
36	0.65077	0.75927	0.85710	1.16672
37	0.65100	0.75908	0.85761	1.16603
38	0.65122	0.75889	0.85811	1.16535
39	0.65144	0.75870	0.85862	1.16466
40	0.65166	0.75851	0.85912	1.16398
41	0.65188	0.75832	0.85963	1.16329
42	0.65210	0.75813	0.86014	1.16261
43	0.65232	0.75794	0.86064	1.16192
44	0.65254	0.75775	0.86115	1.16124
45	0.65276	0.75756	0.86166	1.16056
46	0.65298	0.75738	0.86218	1.15987
47	0.65320	0.75719	0.86267	1.15919
48	0.65342	0.75700	0.86318	1.15851
49	0.65364	0.75680	0.86368	1.15783
50	0.65386	0.75661	0.86419	1.15715
51	0.65408	0.75642	0.86470	1.15647
52	0.65430	0.75623	0.86521	1.15579
53	0.65452	0.75604	0.86572	1.15511
54	0.65474	0.75585	0.86623	1.15443
55	0.65496	0.75566	0.86674	1.15375
56	0.65518	0.75547	0.86725	1.15308
57	0.65540	0.75528	0.86776	1.15240
58	0.65562	0.75509	0.86827	1.15172
59	0.65584	0.75490	0.86878	1.15104
60	0.65606	0.75471	0.86929	1.15037

Cos	Sin	Cot	Tan	Minutes
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41 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.65606	0.75471	0.86929	1.15037	69
1	0.65628	0.75452	0.86900	1.14969	59
2	0.65650	0.75433	0.87031	1.14902	58
3	0.65672	0.75414	0.87082	1.14834	57
4	0.65694	0.75395	0.87133	1.14767	56
5	0.65716	0.75375	0.87184	1.14699	55
6	0.65738	0.75356	0.87236	1.14632	54
7	0.65759	0.75337	0.87287	1.14565	53
8	0.65781	0.75318	0.87338	1.14498	52
9	0.65803	0.75299	0.87389	1.14430	51
10	0.65825	0.75280	0.87441	1.14363	50
11	0.65847	0.75261	0.87492	1.14296	49
12	0.65868	0.75241	0.87543	1.14229	48
13	0.65891	0.75222	0.87595	1.14162	47
14	0.65913	0.75203	0.87646	1.14095	46
15	0.65935	0.75184	0.87698	1.14028	45
16	0.65956	0.75165	0.87749	1.13961	44
17	0.65978	0.75146	0.87801	1.13894	43
18	0.66000	0.75126	0.87852	1.13826	42
19	0.66022	0.75107	0.87904	1.13761	41
20	0.66044	0.75088	0.87955	1.13694	40
21	0.66066	0.75069	0.88007	1.13627	39
22	0.66088	0.75050	0.88059	1.13561	38
23	0.66109	0.75030	0.88110	1.13494	37
24	0.66131	0.75011	0.88162	1.13428	36
25	0.66153	0.74992	0.88214	1.13361	35
26	0.66175	0.74973	0.88265	1.13295	34
27	0.66197	0.74953	0.88317	1.13228	33
28	0.66218	0.74934	0.88369	1.13162	32
29	0.66240	0.74915	0.88421	1.13096	31
30	0.66262	0.74896	0.88473	1.13029	30
31	0.66284	0.74876	0.88524	1.12963	29
32	0.66306	0.74857	0.88576	1.12897	28
33	0.66327	0.74838	0.88628	1.12831	27
34	0.66349	0.74818	0.88680	1.12765	26
35	0.66371	0.74799	0.88732	1.12699	25
36	0.66393	0.74780	0.88784	1.12633	24
37	0.66414	0.74760	0.88836	1.12567	23
38	0.66436	0.74741	0.88888	1.12501	22
39	0.66458	0.74722	0.88940	1.12435	21
40	0.66480	0.74703	0.88992	1.12369	20
41	0.66501	0.74683	0.89045	1.12303	19
42	0.66523	0.74664	0.89097	1.12236	18
43	0.66545	0.74644	0.89149	1.12172	17
44	0.66566	0.74625	0.89201	1.12106	16
45	0.66588	0.74606	0.89253	1.12041	15
46	0.66610	0.74586	0.89305	1.11975	14
47	0.66632	0.74567	0.89358	1.11909	13
48	0.66653	0.74548	0.89410	1.11844	12
49	0.66675	0.74528	0.89463	1.11778	11
50	0.66697	0.74509	0.89515	1.11713	10
51	0.66718	0.74489	0.89567	1.11648	9
52	0.66740	0.74470	0.89620	1.11582	8
53	0.66762	0.74451	0.89672	1.11517	7
54	0.66783	0.74431	0.89725	1.11452	6
55	0.66805	0.74412	0.89777	1.11387	5
56	0.66827	0.74392	0.89830	1.11321	4
57	0.66848	0.74373	0.89883	1.11256	3
58	0.66870	0.74353	0.89935	1.11191	2
59	0.66891	0.74334	0.89988	1.11126	1
60	0.66913	0.74314	0.90040	1.11061	0

Cos Sin Cot Tan Minutes

41 Degrees

42 Degrees

Minutes	Sin	Cos	Tan	Cot	
0	0.66913	0.74314	0.90040	1.11061	60
1	0.66935	0.74295	0.90093	1.10996	59
2	0.66956	0.74276	0.90146	1.10931	58
3	0.66978	0.74256	0.90199	1.10867	57
4	0.66999	0.74237	0.90251	1.10802	56
5	0.67021	0.74217	0.90304	1.10737	55
6	0.67043	0.74198	0.90357	1.10672	54
7	0.67064	0.74178	0.90410	1.10607	53
8	0.67086	0.74159	0.90463	1.10543	52
9	0.67107	0.74139	0.90516	1.10478	51
10	0.67129	0.74120	0.90569	1.10414	50
11	0.67151	0.74100	0.90621	1.10349	49
12	0.67172	0.74080	0.90674	1.10285	48
13	0.67194	0.74061	0.90727	1.10220	47
14	0.67215	0.74041	0.90781	1.10156	46
15	0.67237	0.74022	0.90834	1.10091	45
16	0.67258	0.74002	0.90887	1.10027	44
17	0.67280	0.73983	0.90940	1.09963	43
18	0.67301	0.73963	0.90993	1.09899	42
19	0.67323	0.73944	0.91046	1.09834	41
20	0.67344	0.73924	0.91099	1.09770	40
21	0.67366	0.73904	0.91153	1.09706	39
22	0.67387	0.73885	0.91206	1.09642	38
23	0.67409	0.73865	0.91259	1.09578	37
24	0.67430	0.73846	0.91313	1.09514	36
25	0.67452	0.73826	0.91366	1.09450	35
26	0.67473	0.73806	0.91419	1.09386	34
27	0.67495	0.73787	0.91473	1.09322	33
28	0.67516	0.73767	0.91526	1.09258	32
29	0.67538	0.73747	0.91580	1.09195	31
30	0.67559	0.73728	0.91633	1.09131	30
31	0.67580	0.73708	0.91687	1.09067	29
32	0.67602	0.73688	0.91740	1.09003	28
33	0.67623	0.73669	0.91794	1.08940	27
34	0.67645	0.73649	0.91847	1.08876	26
35	0.67666	0.73629	0.91901	1.08813	25
36	0.67688	0.73610	0.91955	1.08749	24
37	0.67709	0.73590	0.92008	1.08686	23
38	0.67730	0.73570	0.92062	1.08622	22
39	0.67752	0.73551	0.92116	1.08559	21
40	0.67773	0.73531	0.92170	1.08496	20
41	0.67795	0.73511	0.92224	1.08432	19
42	0.67816	0.73491	0.92277	1.08369	18
43	0.67837	0.73472	0.92331	1.08306	17
44	0.67859	0.73452	0.92385	1.08243	16
45	0.67880	0.73432	0.92439	1.08179	15
46	0.67901	0.73413	0.92493	1.08116	14
47	0.67923	0.73393	0.92547	1.08053	13
48	0.67944	0.73373	0.92601	1.07990	12
49	0.67965	0.73353	0.92655	1.07927	11
50	0.67987	0.73333	0.92709	1.07864	10
51	0.68008	0.73314	0.92763	1.07801	9
52	0.68029	0.73294	0.92817	1.07738	8
53	0.68051	0.73274	0.92872	1.07676	7
54	0.68072	0.73254	0.92926	1.07613	6
55	0.68093	0.73234	0.92980	1.07550	5
56	0.68115	0.73215	0.93034	1.07487	4
57	0.68136	0.73195	0.93088	1.07425	3
58	0.68157	0.73175	0.93143	1.07362	2
59	0.68179	0.73155	0.93197	1.07299	1
60	0.68200	0.73135	0.93252	1.07237	0

Cos Sin Cot Tan

47 Degrees

Minutes

43 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.68200	0.73135	0.93252	1.07237	60
1	0.68221	0.73116	0.93306	1.07174	59
2	0.68242	0.73096	0.93360	1.07112	58
3	0.68264	0.73076	0.93415	1.07049	57
4	0.68285	0.73056	0.93469	1.06987	56
5	0.68306	0.73036	0.93524	1.06925	55
6	0.68327	0.73016	0.93578	1.06862	54
7	0.68349	0.72996	0.93633	1.06800	53
8	0.68370	0.72976	0.93688	1.06738	52
9	0.68391	0.72957	0.93742	1.06676	51
10	0.68412	0.72937	0.93797	1.06613	50
11	0.68434	0.72917	0.93852	1.06551	49
12	0.68455	0.72897	0.93906	1.06489	48
13	0.68476	0.72877	0.93961	1.06427	47
14	0.68497	0.72857	0.94016	1.06365	46
15	0.68518	0.72837	0.94071	1.06303	45
16	0.68539	0.72817	0.94125	1.06241	44
17	0.68561	0.72797	0.94180	1.06179	43
18	0.68582	0.72777	0.94235	1.06117	42
19	0.68603	0.72757	0.94290	1.06056	41
20	0.68624	0.72737	0.94345	1.05994	40
21	0.68645	0.72717	0.94400	1.05932	39
22	0.68666	0.72697	0.94455	1.05870	38
23	0.68688	0.72677	0.94510	1.05809	37
24	0.68709	0.72657	0.94565	1.05747	36
25	0.68730	0.72637	0.94620	1.05685	35
26	0.68751	0.72617	0.94676	1.05624	34
27	0.68772	0.72597	0.94731	1.05562	33
28	0.68793	0.72577	0.94786	1.05501	32
29	0.68814	0.72557	0.94841	1.05439	31
30	0.68835	0.72537	0.94896	1.05378	30
31	0.68857	0.72517	0.94952	1.05317	29
32	0.68878	0.72497	0.95007	1.05255	28
33	0.68899	0.72477	0.95062	1.05194	27
34	0.68920	0.72457	0.95118	1.05133	26
35	0.68941	0.72437	0.95173	1.05072	25
36	0.68962	0.72417	0.95229	1.05010	24
37	0.68983	0.72397	0.95284	1.04949	23
38	0.69004	0.72377	0.95340	1.04888	22
39	0.69025	0.72357	0.95395	1.04827	21
40	0.69046	0.72337	0.95451	1.04766	20
41	0.69067	0.72317	0.95506	1.04705	19
42	0.69088	0.72297	0.95562	1.04644	18
43	0.69109	0.72277	0.95618	1.04583	17
44	0.69130	0.72257	0.95673	1.04522	16
45	0.69151	0.72236	0.95729	1.04461	15
46	0.69172	0.72216	0.95785	1.04401	14
47	0.69193	0.72196	0.95841	1.04340	13
48	0.69214	0.72176	0.95897	1.04279	12
49	0.69235	0.72156	0.95952	1.04218	11
50	0.69256	0.72136	0.96008	1.04158	10
51	0.69277	0.72116	0.96064	1.04097	9
52	0.69298	0.72095	0.96120	1.04036	8
53	0.69319	0.72075	0.96176	1.03976	7
54	0.69340	0.72055	0.96232	1.03915	6
55	0.69361	0.72035	0.96288	1.03855	5
56	0.69382	0.72015	0.96344	1.03794	4
57	0.69403	0.71995	0.96400	1.03734	3
58	0.69424	0.71974	0.96457	1.03674	2
59	0.69445	0.71954	0.96513	1.03613	1
60	0.69466	0.71934	0.96569	1.03553	0

Cos

Sin

Cot

Tan

Minutes

44 Degree

Minutes	Sin	Cos	Tan	Cot	
0	0.69466	0.71934	0.96569	1.03553	60
1	0.69487	0.71914	0.96625	1.03493	59
2	0.69508	0.71894	0.96681	1.03433	58
3	0.69529	0.71873	0.96738	1.03372	57
4	0.69549	0.71853	0.96794	1.03312	56
5	0.69570	0.71833	0.96850	1.03252	55
6	0.69591	0.71813	0.96907	1.03192	54
7	0.69612	0.71792	0.96963	1.03132	53
8	0.69633	0.71772	0.97020	1.03072	52
9	0.69654	0.71752	0.97076	1.03012	51
10	0.69675	0.71732	0.97133	1.02952	50
11	0.69696	0.71711	0.97189	1.02892	49
12	0.69717	0.71691	0.97246	1.02832	48
13	0.69737	0.71671	0.97302	1.02772	47
14	0.69758	0.71650	0.97359	1.02713	46
15	0.69779	0.71630	0.97416	1.02653	45
16	0.69800	0.71610	0.97472	1.02593	44
17	0.69821	0.71590	0.97529	1.02533	43
18	0.69842	0.71569	0.97586	1.02474	42
19	0.69862	0.71549	0.97643	1.02414	41
20	0.69883	0.71529	0.97700	1.02355	40
21	0.69904	0.71508	0.97756	1.02295	39
22	0.69925	0.71488	0.97813	1.02236	38
23	0.69946	0.71468	0.97870	1.02176	37
24	0.69966	0.71447	0.97927	1.02117	36
25	0.69987	0.71427	0.97984	1.02057	35
26	0.70008	0.71407	0.98041	1.01998	34
27	0.70029	0.71386	0.98098	1.01939	33
28	0.70049	0.71366	0.98155	1.01879	32
29	0.70070	0.71345	0.98213	1.01820	31
30	0.70091	0.71325	0.98270	1.01761	30
31	0.70112	0.71305	0.98327	1.01702	29
32	0.70132	0.71284	0.98384	1.01642	28
33	0.70153	0.71264	0.98441	1.01583	27
34	0.70174	0.71243	0.98499	1.01524	26
35	0.70195	0.71223	0.98556	1.01465	25
36	0.70215	0.71203	0.98613	1.01406	24
37	0.70236	0.71182	0.98671	1.01347	23
38	0.70257	0.71162	0.98728	1.01288	22
39	0.70277	0.71141	0.98786	1.01229	21
40	0.70298	0.71121	0.98843	1.01170	20
41	0.70319	0.71100	0.98901	1.01112	19
42	0.70339	0.71080	0.98958	1.01053	18
43	0.70360	0.71059	0.99016	1.00994	17
44	0.70381	0.71039	0.99073	1.00935	16
45	0.70401	0.71019	0.99131	1.00876	15
46	0.70422	0.70998	0.99189	1.00818	14
47	0.70443	0.70978	0.99247	1.00759	13
48	0.70463	0.70957	0.99304	1.00701	12
49	0.70484	0.70937	0.99362	1.00642	11
50	0.70505	0.70916	0.99420	1.00583	10
51	0.70525	0.70896	0.99478	1.00525	9
52	0.70546	0.70875	0.99536	1.00467	8
53	0.70567	0.70855	0.99594	1.00408	7
54	0.70587	0.70834	0.99652	1.00350	6
55	0.70608	0.70813	0.99710	1.00291	5
56	0.70628	0.70793	0.99768	1.00233	4
57	0.70649	0.70772	0.99826	1.00175	3
58	0.70670	0.70752	0.99884	1.00116	2
59	0.70690	0.70731	0.99942	1.00058	1
60	0.70711	0.70711	1.00000	1.00000	0

Cos Sin Cot Tan Minutes

45 Degrees

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APPENDIX D
ENGLISH/METRIC CONVERSION CHART

To Convert	into	Multiply By
Feet	meters	0.3048
Inches	centimeters	2.54
Pounds	kilograms	0.4536

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